LESSON 1-5

Equations

Then	Now	Why?
• You simplified expressions.	• 1 Solve equations with one variable. 2 Solve equations with two variables.	• Mark's baseball team scored 3 runs in the first inning. At the top of the third inning, their score was 4. The open sentence below represents the change in their score. $3 + r = 4$ The solution is 1. The team got 1 run in the second inning.

New Vocabulary
open sentence
equation
solving
solution
replacement set
set
element
solution set
identity

Common Core State Standards

Content Standards
A.CED.1 Create equations and inequalities in one variable and use them to solve problems.
A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematical Practices
3 Construct viable arguments and critique the reasoning of others.

1 Solve Equations

A mathematical statement that contains algebraic expressions and symbols is an **open sentence**. A sentence that contains an equals sign, =, is an **equation**.

expression → $3x + 7$ $\qquad$ $3x + 7 = 13$ ← equation

Finding a value for a variable that makes a sentence true is called **solving** the open sentence. This replacement value is a **solution**.

A set of numbers from which replacements for a variable may be chosen is called a **replacement set**. A **set** is a collection of objects or numbers that is often shown using braces. Each object or number in the set is called an **element**, or member. A **solution set** is the set of elements from the replacement set that make an open sentence true.

Example 1 Use a Replacement Set

Find the solution set of the equation $2q + 5 = 13$ if the replacement set is {2, 3, 4, 5, 6}.

Use a table to solve. Replace q in $2q + 5 = 13$ with each value in the replacement set.

q	$2q + 5 = 13$	True or False?
2	$2(2) + 5 = 13$	false
3	$2(3) + 5 = 13$	false
4	$2(4) + 5 = 13$	true
5	$2(5) + 5 = 13$	false
6	$2(6) + 5 = 13$	false

Since the equation is true when $q = 4$, the solution of $2q + 5 = 13$ is $q = 4$.

The solution set is {4}.

Guided Practice

Find the solution set for each equation if the replacement set is {0, 1, 2, 3}.

1A. $8m - 7 = 17$ $\qquad$ **1B.** $28 = 4(1 + 3d)$

Paul Rees/ImageState

You can often solve an equation by applying the order of operations.

Standardized Test Example 2 Apply the Order of Operations

Solve $6 + (5^2 - 5) \div 2 = p$.

A 3 B 6 C 13 D 16

Test-Taking Tip

Rewrite the Equation

If you are allowed to write in your testing booklet, it can be helpful to rewrite the equation with simplified terms.

Read the Test Item

You need to apply the order of operations to the expression in order to solve for p.

Solve the Test Item

$6 + (5^2 - 5) \div 2 = p$ Original equation

$6 + (25 - 5) \div 2 = p$ Evaluate powers.

$6 + 20 \div 2 = p$ Subtract 5 from 25.

$6 + 10 = p$ Divide 20 by 2.

$16 = p$ Add.

The correct answer is D.

Guided Practice

2. Solve $t = 9^2 \div (5 - 2)$.

F 3 G 6 H 14.2 J 27

Some equations have a unique solution. Other equations do not have a solution.

Example 3 Solutions of Equations

Solve each equation.

Study Tip

Guess and Check When the solution to an equation is not easy to see, substitute values for x and test the equation. Continue to test values until you get a true statement. For example, if $3x + 16 = 73$, test values for x.

$3(10) + 16 = 48$ too low

$3(20) + 16 = 76$ too high

$3(19) + 16 = 73$ ✓

a. $7 - (4^2 - 10) + n = 10$

Simplify the equation first and then look for a solution.

$7 - (4^2 - 10) + n = 10$ Original equation

$7 - (16 - 10) + n = 10$ Evaluate powers.

$7 - 6 + n = 10$ Subtract 10 from 16.

$1 + n = 10$ Subtract 6 from 7.

The only value for n that makes the equation true is 9. Therefore, this equation has a unique solution of 9.

b. $n(3 + 2) + 6 = 5n + (10 - 3)$

$n(3 + 2) + 6 = 5n + (10 - 3)$ Original equation

$n(5) + 6 = 5n + (10 - 3)$ Add 3 + 2.

$n(5) + 6 = 5n + 7$ Subtract 3 from 10.

$5n + 6 = 5n + 7$ Commutative (×)

No matter what real value is substituted for n, the left side of the equation will always be one less than the right side. So, the equation will never be true. Therefore, there is no solution of this equation.

Guided Practice

3A. $(18 + 4) + m = (5 - 3)m$ **3B.** $8 \cdot 4 \cdot k + 9 \cdot 5 = (36 - 4)k - (2 \cdot 5)$

ReadingMath

Identities An identity is an equation that shows that a number or expression is equivalent to itself.

An equation that is true for every value of the variable is called an **identity**.

Example 4 Identities

Solve $(2 \cdot 5 - 8)(3h + 6) = [(2h + h) + 6]2$.

$(2 \cdot 5 - 8)(3h + 6) = [(2h + h) + 6]2$		Original Equation
$(10 - 8)(3h + 6) = [(2h + h) + 6]2$		Multiply $2 \cdot 5$.
$2(3h + 6) = [(2h + h) + 6]2$		Subtract 8 from 10.
$6h + 12 = [(2h + h) + 6]2$		Distributive Property
$6h + 12 = [3h + 6]2$		Add $2h + h$.
$6h + 12 = 6h + 12$		Distributive Property

No matter what value is substituted for h, the left side of the equation will always be equal to the right side. So, the equation will always be true. Therefore, the solution of this equation could be any real number.

GuidedPractice

Solve each equation.

4A. $12(10 - 7) + 9g = g(2^2 + 5) + 36$

4B. $2d + (2^3 - 5) = 10(5 - 2) + d(12 \div 6)$

4C. $3(b + 1) - 5 = 3b - 2$

4D. $5 - \frac{1}{2}(c - 6) = 4$

2 Solve Equations with Two Variables

Some equations contain two variables. It is often useful to make a table of values and use substitution to find the corresponding values of the second variable.

Example 5 Equations Involving Two Variables

MOVIE RENTALS **Mr. Hernandez pays $10 each month for movies delivered by mail. He can also rent movies in the store for $1.50 per title. Write and solve an equation to find the total amount Mr. Hernandez spends this month if he rents 3 movies from the store.**

The cost of the movie plan is a flat rate. The variable is the number of movies he rents from the store. The total cost is the price of the plan plus $1.50 times the number of movies from the store. Let C be the total cost and m be the number of movies.

$C = 1.50m + 10$	Original equation
$= 1.50(3) + 10$	Substitute 3 for m.
$= 4.50 + 10$	Multiply.
$= 14.50$	

Mr. Hernandez spends $14.50 on movie rentals in one month.

GuidedPractice

5. TRAVEL Amelia drives an average of 65 miles per hour. Write and solve an equation to find the time it will take her to drive 36 miles.

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1 **Find the solution set of each equation if the replacement set is {11, 12, 13, 14, 15}.**

1. $n + 10 = 23$

2. $7 = \frac{c}{2}$

3. $29 = 3x - 7$

4. $(k - 8)12 = 84$

Example 2 **5. MULTIPLE CHOICE** Solve $\frac{d + 5}{10} = 2$.

A 10 B 15 C 20 D 25

Examples 3–4 **Solve each equation.**

6. $x = 4(6) + 3$

7. $14 - 82 = w$

8. $5 + 22a = 2 + 10 \div 2$

9. $(2 \cdot 5) + \frac{c^3}{3} = c^3 \div (1^5 + 2) + 10$

Example 5 **10. RECYCLING** San Francisco has a recycling facility that accepts unused paint. Volunteers blend and mix the paint and give it away in 5-gallon buckets. Write and solve an equation to find the number of buckets of paint given away from the 30,000 gallons that are donated.

Practice and Problem Solving

Extra Practice is on page R1.

Example 1 **Find the solution set of each equation if the replacement sets are y: {1, 3, 5, 7, 9} and z: {10, 12, 14, 16, 18}.**

11. $z + 10 = 22$

12. $52 = 4z$

13. $\frac{15}{y} = 3$

14. $17 = 24 - y$

15. $2z - 5 = 27$

16. $4(y + 1) = 40$

17. $22 = \frac{60}{y} + 2$

18. $111 = z^2 + 11$

Examples 2–4 **Solve each equation.**

19. $a = 32 - 9(2)$

20. $w = 56 \div (2^2 + 3)$

21. $\frac{27 + 5}{16} = g$

22. $\frac{12 \cdot 5}{15 - 3} = y$

23. $r = \frac{9(6)}{(8 + 1)3}$

24. $a = \frac{4(14 - 1)}{3(6) - 5} + 7$

25. $(4 - 2^2 + 5)w = 25$

26. $7 + x - (3 + 32 \div 8) = 3$

27. $3^2 - 2 \cdot 3 + u = (3^3 - 3 \cdot 8)(2) + u$

28. $(3 \cdot 6 \div 2)v + 10 = 3^2v + 9$

29. $6k + (3 \cdot 10 - 8) = (2 \cdot 3)k + 22$

30. $(3 \cdot 5)t + (21 - 12) = 15t + 3^2$

31. $(2^4 - 3 \cdot 5)q + 13 = (2 \cdot 9 - 4^2)q + \left(\frac{3 \cdot 4}{12} - 1\right)$

32. $\frac{3 \cdot 22}{18 + 4}r - \left(\frac{4^2}{9 + 7} - 1\right) = r + \left(\frac{8 \cdot 9}{3} \div 3\right)$

33. SCHOOL A conference room can seat a maximum of 85 people. The principal and two counselors need to meet with the school's juniors to discuss college admissions. If each student must bring a parent with them, how many students can attend each meeting? Assume that each student has a unique set of parents.

34. CCSS MODELING The perimeter of a regular octagon is 128 inches. Find the length of each side.

Example 5

35 **SPORTS** A 200-pound athlete who trains for four hours per day requires 2836 Calories for basic energy requirements. During training, the same athlete requires an additional 3091 Calories for extra energy requirements. Write an equation to find C, the total daily Calorie requirement for this athlete. Then solve the equation.

36. **ENERGY** An electric generator can power 3550 watts of electricity. Write and solve an equation to find how many 75-watt light bulbs a generator could power.

Make a table of values for each equation if the replacement set is {−2, −1, 0, 1, 2}.

37. $y = 3x - 2$

38. $3.25x + 0.75 = y$

Solve each equation using the given replacement set.

39. $t - 13 = 7$; {10, 13, 17, 20}

40. $14(x + 5) = 126$; {3, 4, 5, 6, 7}

41. $22 = \frac{n}{3}$; {62, 64, 66, 68, 70}

42. $35 = \frac{g - 8}{2}$; {78, 79, 80, 81}

Solve each equation.

43. $\frac{3(9) - 2}{1 + 4} = d$

44. $j = 15 \div 3 \cdot 5 - 4^2$

45. $c + (3^2 - 3) = 21$

46. $(3^3 - 3 \cdot 9) + (7 - 2^2)b = 24b$

47. CCSS **SENSE-MAKING** Blood flow rate can be expressed as $F = \frac{p_1 - p_2}{r}$, where F is the flow rate, p_1 and p_2 are the initial and final pressure exerted against the blood vessel's walls, respectively, and r is the resistance created by the size of the vessel.

a. Write and solve an equation to determine the resistance of the blood vessel for an initial pressure of 100 millimeters of mercury, a final pressure of 0 millimeters of mercury, and a flow rate of 5 liters per minute.

b. Use the equation to complete the table below.

Initial Pressure p_1 (mm Hg)	Final Pressure p_2 (mm Hg)	Resistance r (mm Hg/L/min)	Blood Flow Rate F (L/min)
100	0		5
100	0	30	
	5	40	4
90		10	6

Determine whether the given number is a solution of the equation.

48. $x + 6 = 15$; 9

49. $12 + y = 26$; 14

50. $2t - 10 = 4$; 3

51. $3r + 7 = -5$; 2

52. $6 + 4m = 18$; 3

53. $-5 + 2p = -11$; −3

54. $\frac{q}{2} = 20$; 10

55. $\frac{w - 4}{5} = -3$; −11

56. $\frac{g}{3} - 4 = 12$; 48

Make a table of values for each equation if the replacement set is {−2, −1, 0, 1, 2}.

57. $y = 3x + 5$

58. $-2x - 3 = y$

59. $y = \frac{1}{2}x + 2$

60. $4.2x - 1.6 = y$

61. **GEOMETRY** The length of a rectangle is 2 inches greater than the width. The length of the base of an isosceles triangle is 12 inches, and the lengths of the other two sides are 1 inch greater than the width of the rectangle.

a. Draw a picture of each figure and label the dimensions.

b. Write two expressions to find the perimeters of the rectangle and triangle.

c. Find the width of the rectangle if the perimeters of the figures are equal.

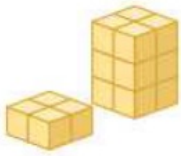
62. CONSTRUCTION The construction of a building requires 10 tons of steel per story.

a. Define a variable and write an equation for the number of tons of steel required if the building has 15 stories.

b. How many tons of steel are needed?

63. MULTIPLE REPRESENTATIONS In this problem, you will further explore writing equations.

a. Concrete Use centimeter cubes to build a tower similar to the one shown at the right.



b. Tabular Copy and complete the table shown below. Record the number of layers in the tower and the number of cubes used in the table.

Layers	1	2	3	4	5	6	7
Cubes	?	?	?	?	?	?	?

c. Analytical As the number of layers in the tower increases, how does the number of cubes in the tower change?

d. Algebraic Write a rule that gives the number of cubes in terms of the number of layers in the tower.

H.O.T. Problems Use Higher-Order Thinking Skills

64. REASONING Compare and contrast an expression and an equation.

65. OPEN ENDED Write an equation that is an identity.

66. REASONING Explain why an open sentence always has at least one variable.

67. CCSS CRITIQUE Tom and Li-Cheng are solving the equation $x = 4(3 - 2) + 6 \div 8$. Is either of them correct? Explain your reasoning.

Tom

$$x = 4(3-2) + 6 \div 8$$
$$= 4(1) + 6 \div 8$$
$$= 4 + 6 \div 8$$
$$= 4 + \frac{6}{8}$$
$$= 4\frac{3}{4}$$

Li-Cheng

$$x = 4(3-2) + 6 \div 8$$
$$= 4(1) + 6 \div 8$$
$$= 4 + 6 \div 8$$
$$= 10 \div 8$$
$$= \frac{5}{4}$$

68. CHALLENGE Find all of the solutions of $x^2 + 5 = 30$.

69. OPEN ENDED Write an equation that involves two or more operations with a solution of -7.

70. WRITING IN MATH Explain how you can determine that an equation has no real numbers as a solution. How can you determine that an equation has all real numbers as solutions?

Standardized Test Practice

71. Which of the following is *not* an equation?

A $y = 6x - 4$

B $\frac{a+4}{2} = \frac{1}{4}$

C $(4 \cdot 3b) + (8 \div 2c)$

D $55 = 6 + d^2$

72. SHORT RESPONSE The expected attendance for the Drama Club production is 65% of the student body. If the student body consists of 300 students, how many students are expected to attend?

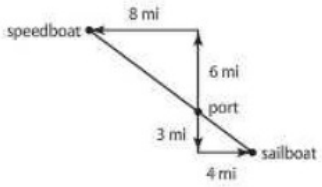
73. GEOMETRY A speedboat and a sailboat take off from the same port. The diagram shows their travel. What is the distance between the boats?

F 12 mi

G 15 mi

H 18 mi

J 24 mi



74. Michelle can read 1.5 pages per minute. How many pages can she read in two hours?

A 90 pages

B 120 pages

C 150 pages

D 180 pages

Spiral Review

75. ZOO A zoo has about 500 children and 750 adults visit each day. Write an expression to represent about how many visitors the zoo will have over a month. (Lesson 1-4)

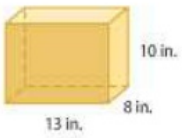
Find the value of p in each equation. Then name the property that is used. (Lesson 1-3)

76. $7.3 + p = 7.3$

77. $12p = 1$

78. $1p = 4$

79. MOVING BOXES The figure shows the dimensions of the boxes Steve uses to pack. How many cubic inches can each box hold? (Lesson 0-9)



Express each percent as a fraction. (Lesson 0-6)

80. 35%

81. 15%

82. 28%

For each problem, determine whether you need an estimate or an exact answer. Then solve. (Lessons 0-6 and 0-1)

83. TRAVEL The distance from Raleigh, North Carolina, to Philadelphia, Pennsylvania, is approximately 428 miles. The average gas mileage of José's car is 45 miles per gallon. About how many gallons of gas will be needed to make the trip?

84. PART-TIME JOB An employer pays $8.50 per hour. If 20% of pay is withheld for taxes, what are the take-home earnings from 28 hours of work?

Skills Review

Find each sum or difference.

85. $1.14 + 5.6$

86. $4.28 - 2.4$

87. $8 - 6.35$

88. $\frac{4}{5} + \frac{1}{6}$

89. $\frac{2}{7} + \frac{3}{4}$

90. $\frac{6}{8} - \frac{1}{2}$