LESSON 1-8 Interpreting Graphs of Functions

Then

- You identified functions and found function values.

Now

1. Interpret intercepts, and symmetry of graphs of functions.
2. Interpret positive, negative, increasing, and decreasing behavior, extrema, and end behavior of graphs of functions.

Why?

- Sales of video games, including hardware, software, and accessories, have increased at times and decreased at other times over the years. Annual retail video game sales in the U.S. from 2000 to 2009 can be modeled by the graph of a nonlinear function.

New Vocabulary

intercept
x-intercept
y-intercept
line symmetry
positive
negative
increasing
decreasing
extrema
relative maximum
relative minimum
end behavior

Common Core State Standards

Content Standards

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Mathematical Practices

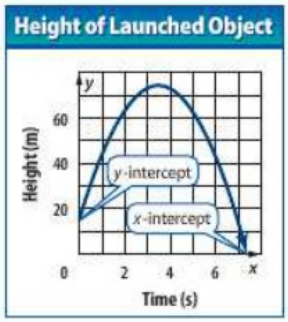
1 Make sense of problems and persevere in solving them.

1 Interpret Intercepts and Symmetry

To interpret the graph of a function, estimate and interpret key features. The **intercepts** of a graph are points where the graph intersects an axis. The y-coordinate of the point at which the graph intersects the y-axis is called a **y-intercept**. Similarly, the x-coordinate of the point at which a graph intersects the x-axis is called an **x-intercept**.

Real-World Example 1 Interpret Intercepts

PHYSICS The graph shows the height y of an object as a function of time x. Identify the function as *linear* or *nonlinear*. Then estimate and interpret the intercepts.



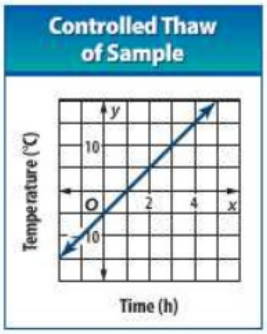
Linear or Nonlinear: Since the graph is a curve and not a line, the graph is nonlinear.

y-Intercept: The graph intersects the y-axis at about (0, 15), so the y-intercept of the graph is about 15. This means that the object started at an initial height of about 15 meters above the ground.

x-Intercept(s): The graph intersects the x-axis at about (7.4, 0), so the x-intercept is about 7.4. This means that the object struck the ground after about 7.4 seconds.

Guided Practice

1. The graph shows the temperature y of a medical sample thawed at a controlled rate. Identify the function as *linear* or *nonlinear*. Then estimate and interpret the intercepts.

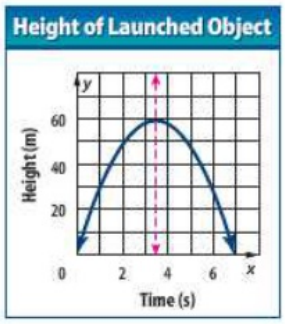


Kiyoshi Ota/Bloomberg/Getty Images

The graphs of some functions exhibit another key feature: symmetry. A graph possesses **line symmetry** in the y-axis or some other vertical line if each half of the graph on either side of the line matches exactly.

Real-World Example 2 Interpret Symmetry

PHYSICS An object is launched. The graph shows the height y of the object as a function of time x. Describe and interpret any symmetry.



The right half of the graph is the mirror image of the left half in approximately the line $x = 3.5$ between approximately $x = 0$ and $x = 7$.

In the context of the situation, the symmetry of the graph tells you that the time it took the object to go up is equal to the time it took to come down.

Guided Practice

2. Describe and interpret any symmetry exhibited by the graph in Guided Practice 1.

Study Tip

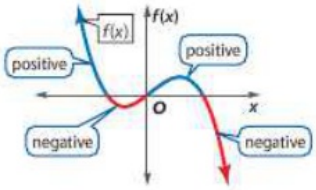
Symmetry The graphs of most real-world functions do not exhibit symmetry over the entire domain. However, many have symmetry over smaller portions of the domain that are worth analyzing.

2 Interpret Extrema and End Behavior

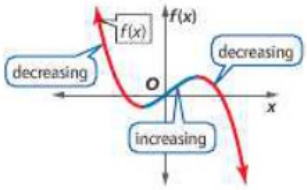
Interpreting a graph also involves estimating and interpreting where the function is increasing, decreasing, positive, or negative, and where the function has any extreme values, either high or low.

Key Concepts Positive, Negative, Increasing, Decreasing, Extrema, and End Behavior

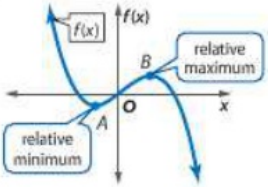
A function is **positive** where its graph lies *above* the x-axis, and **negative** where its graph lies *below* the x-axis.



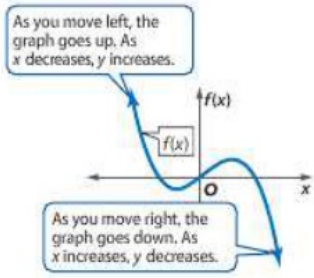
A function is **increasing** where the graph goes *up* and **decreasing** where the graph goes *down* when viewed from left to right.



The points shown are the locations of relatively high or low function values called **extrema**. Point A is a **relative minimum**, since no other nearby points have a lesser y-coordinate. Point B is a **relative maximum**, since no other nearby points have a greater y-coordinate.



End behavior describes the values of a function at the positive and negative extremes in its domain.



Study Tip

End Behavior The end behavior of some graphs can be described as approaching a specific y-value. In this case, a portion of the graph looks like a horizontal line.

Real-World Link

The first successful commercially sold portable video game system was released in 1989 and sold for $120.

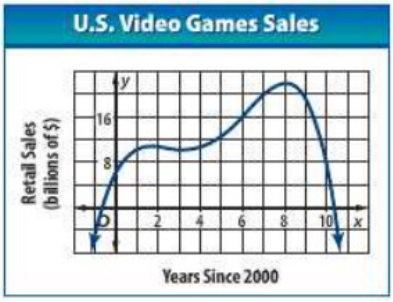
Source: *PCWorld*

Study Tip

Constant A function is *constant* where the graph does not go up or down as the graph is viewed from left to right.

Real-World Example 3 Interpret Extrema and End Behavior

VIDEO GAMES U.S. retail sales of video games from 2000 to 2009 can be modeled by the function graphed at the right. Estimate and interpret where the function is positive, negative, increasing, and decreasing, the x-coordinates of any relative extrema, and the end behavior of the graph.



Positive: between about $x = -0.6$ and $x = 10.4$

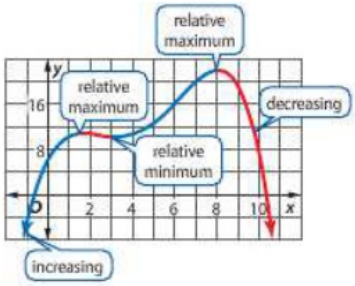
Negative: for about $x < -0.6$ and $x > 10.4$

This means that there were positive sales between about 2000 and 2010, but the model predicts negative sales after about 2010, indicating the unlikely collapse of the industry.

Increasing: for about $x < 1.5$ and between about $x = 3$ and $x = 8$

Decreasing: between about $x = 2$ and $x = 3$ and for about $x > 8$

This means that sales increased from about 2000 to 2002, decreased from 2002 to 2003, increased from 2003 to 2008, and have been decreasing since 2008.



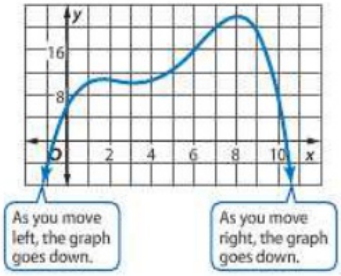
Relative Maximums: at about $x = 1.5$ and $x = 8$

Relative Minimum: at about $x = 3$

The extrema of the graph indicate that the industry experienced two relative peaks in sales during this period: one around 2002 of approximately $10.5 billion and another around 2008 of approximately $22 billion. A relative low of $10 billion in sales came in about 2003.

End Behavior:
As x increases or decreases, the value of y decreases.

The end behavior of the graph indicates negative sales several years prior to 2000 and several years after 2009, which is unlikely. This graph appears to only model sales well between 2000 and 2009 and can only be used to predict sales in 2010.



Guided Practice

3. Estimate and interpret where the function graphed in Guided Practice 1 is positive, negative, increasing, or decreasing, the x-coordinate of any relative extrema, and the end behavior of the graph.

Science & Society Picture Library/SSPL/Getty Images

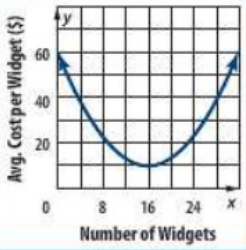
Check Your Understanding

= Step-by-Step Solutions begin on page R13.

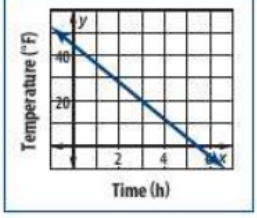
Examples 1–3 CCSS **SENSE-MAKING** Identify the function graphed as *linear* or *nonlinear*. Then estimate and interpret the intercepts of the graph, any symmetry, where the function is positive, negative, increasing, and decreasing, the x-coordinate of any relative extrema, and the end behavior of the graph.

1. **Stock Value**

2. **Average Widget Production Cost**



3. **Temperature Change**

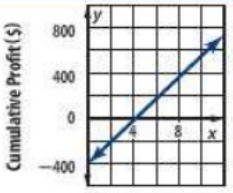


Practice and Problem Solving

Extra Practice is on page R1.

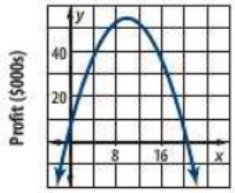
Examples 1–3 CCSS **SENSE-MAKING** Identify the function graphed as *linear* or *nonlinear*. Then estimate and interpret the intercepts of the graph, any symmetry, where the function is positive, negative, increasing, and decreasing, the x-coordinate of any relative extrema, and the end behavior of the graph.

4. **Lawn Mowing Service**

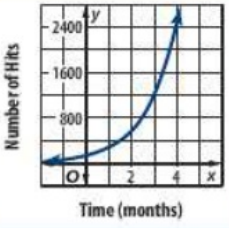


5. **Vehicle Depreciation**

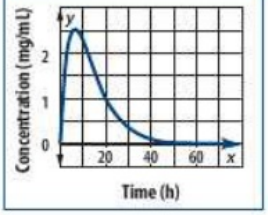
6. **Company Advertising**



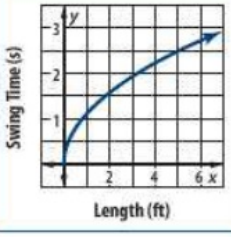
7. **Web Site Traffic**



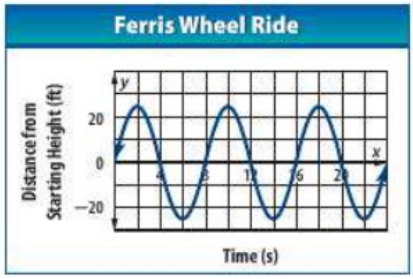
8. **Medicine Concentration**



9. **Pendulum Swing Time**



10. **FERRIS WHEEL** At the beginning of a Ferris wheel ride, a passenger cart is located at the same height as the center of the wheel. The position y in feet of this cart relative to the center t seconds after the ride starts is given by the function graphed at the right. Identify and interpret the key features of the graph. (*Hint:* Look for a pattern in the graph to help you describe its end behavior.)



Sketch a graph of a function that could represent each situation. Identify and interpret the intercepts of the graph, where the graph is increasing and decreasing, and any relative extrema.

11. the height of a corn plant from the time the seed is planted until it reaches maturity 120 days later

12. the height of a football from the time it is punted until it reaches the ground 2.8 seconds later

13. the balance due on a car loan from the date the car was purchased until it was sold 4 years later

Sketch graphs of functions with the following characteristics.

14. The graph is linear with an x-intercept at -2. The graph is positive for $x < -2$, and negative for $x > -2$.

15. A nonlinear graph has x-intercepts at -2 and 2 and a y-intercept at -4. The graph has a relative minimum of -4 at $x = 0$. The graph is decreasing for $x < 0$ and increasing for $x > 0$.

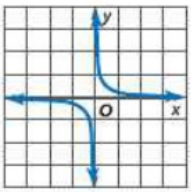
16. A nonlinear graph has a y-intercept at 2, but no x-intercepts. The graph is positive and increasing for all values of x.

17. A nonlinear graph has x-intercepts at -8 and -2 and a y-intercept at 3. The graph has relative minimums at $x = -6$ and $x = 6$ and a relative maximum at $x = 2$. The graph is positive for $x < -8$ and $x > -2$ and negative between $x = -8$ and $x = -2$. As x decreases, y increases and as x increases, y increases.

H.O.T. Problems Use Higher-Order Thinking Skills

18. **CCSS CRITIQUE** Katara thinks that all linear functions have exactly one x-intercept. Desmond thinks that a linear function can have at most one x-intercept. Is either of them correct? Explain your reasoning.

19. **CHALLENGE** Describe the end behavior of the graph shown.



20. **REASONING** Determine whether the following statement is *true* or *false*. Explain.

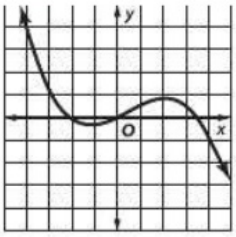
Functions have at most one y-intercept.

21. **OPEN ENDED** Sketch the graph of a function with one relative maximum and one relative minimum that could represent a real-world function. Label each axis and include appropriate units. Then identify and interpret the relative extrema of your graph.

22. **WRITING IN MATH** Describe how you would identify the key features of a graph described in this lesson using a table of values for a function.

Standardized Test Practice

23. Which sentence best describes the end behavior of the function shown?



A As x increases, y increases, and as x decreases, y increases.

B As x increases, y increases, and as x decreases, y decreases.

C As x increases, y decreases, and as x decreases, y increases.

D As x increases, y decreases, and as x decreases, y decreases.

24. Which illustrates the Transitive Property of Equality?

F If $c = 1$, then $c \cdot \frac{1}{c} = 1$.

G If $c = d$ and $d = f$, then $c = f$.

H If $c = d$, then $d = c$.

J If $c = d$ and $d = c$, then $c = 1$.

25. Simplify the expression $5d(7 - 3) - 16d + 3 \cdot 2d$.

A $10d$ **C** $21d$

B $14d$ **D** $25d$

26. What is the probability of selecting a red card or an ace from a standard deck of cards?

F $\frac{1}{26}$ **G** $\frac{1}{2}$ **H** $\frac{7}{13}$ **J** $\frac{15}{26}$

Spiral Review

Determine whether each relation is a function. (Lesson 1-7)

27.

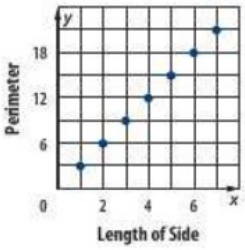
28. $\{(0, 2), (3, 5), (0, -1), (-2, 4)\}$

29.

x	y
17	6
18	6
19	5
20	4

30. **GEOMETRY** Express the relation in the graph at the right as a set of ordered pairs. Describe the domain and range. (Lesson 1-6)

Equilateral Triangles



Use the Distributive Property to rewrite each expression. (Lesson 1-4)

31. $\frac{1}{2}d(2d + 6)$ **32.** $-h(6h - 1)$ **33.** $3z - 6x$

34. **CLOTHING** Robert has 30 socks in his sock drawer. 16 of the socks are white, 6 are black, 2 are red, and 6 are yellow. What is the probability that he randomly pulls out a black sock? (Lesson 0-11)

Skills Review

Evaluate each expression.

35. $(-7)^2$ **36.** 3.2^2 **37.** $(-4.2)^2$ **38.** $\left(\frac{1}{4}\right)^2$