LESSON 2-6 Ratios and Proportions

Then

- You evaluated percents by using a proportion.

Now

1. Compare ratios.
2. Solve proportions.

Why?

- Ratios allow us to compare many items by using a common reference. The table below shows the number of restaurants a certain popular fast food chain has per 10,000 people in the United States as well as other countries. This allows us to compare the number of these restaurants using an equal reference.

Countries	United States	New Zealand	Canada	Australia	Japan	Singapore
Number of Restaurants per 10,000 People	0.433	0.369	0.352	0.349	0.282	0.273

New Vocabulary

ratio
proportion
means
extremes
rate
unit rate
scale
scale model

Common Core State Standards

Content Standards

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematical Practices

6 Attend to precision.

Nico Kaj/Iconica/Getty Images

1 Ratios and Proportions

The comparison between the number of restaurants and the number of people is a ratio. A **ratio** is a comparison of two numbers by division. The ratio of x to y can be expressed in the following ways.

$$x \text{ to } y \qquad x : y \qquad \frac{x}{y}$$

Suppose you wanted to determine the number of restaurants per 100,000 people in Australia. Notice that this ratio is equal to the original ratio.

$$\frac{0.349 \text{ restaurants}}{10{,}000 \text{ people}} = \frac{3.49 \text{ restaurants}}{100{,}000 \text{ people}}$$

$\times 10$ $\times 10$

An equation stating that two ratios are equal is called a **proportion**. So, we can state that $\frac{0.349}{10{,}000} = \frac{3.49}{100{,}000}$ is a proportion.

Example 1 Determine Whether Ratios Are Equivalent

Determine whether $\frac{2}{3}$ and $\frac{16}{24}$ are equivalent ratios. Write *yes* or *no*. Justify your answer.

$$\frac{2}{3} = \frac{2}{3} \qquad \frac{16}{24} = \frac{2}{3}$$

$\div 1$ $\div 1$ $\div 8$ $\div 8$

When expressed in simplest form, the ratios are equivalent.

Guided Practice

Determine whether each pair of ratios are equivalent ratios. Write *yes* or *no*. Justify your answer.

1A. $\frac{6}{10}, \frac{2}{5}$

1B. $\frac{1}{6}, \frac{5}{30}$

StudyTip

Means and Extremes

To solve a proportion using cross products, write an equation that sets the product of the extremes equal to the product of the means.

There are special names for the terms in a proportion.

1.5 and 1.2 are called the **means**. They are the middle terms of the proportion.

$$0.2 : 1.5 = 1.2 : 9.0$$

0.2 and 9.0 are called the **extremes**. They are the first and last terms of the proportion.

KeyConcept **Means-Extremes Property of Proportion**

Words	In a proportion, the product of the extremes is equal to the product of the means.
Symbols	If $\frac{a}{b} = \frac{c}{d}$ and $b, d \neq 0$, then $ad = bc$.
Examples	Since $\frac{2}{4} = \frac{1}{2}$, $2(2) = 4(1)$ or $4 = 4$.

Another way to determine whether two ratios form a proportion is to use cross products. If the cross products are equal, then the ratios form a proportion.

This is the same as multiplying the means, and multiplying the extremes.

Example 2 **Cross Products**

Use cross products to determine whether each pair of ratios forms a proportion.

a. $\frac{2}{3.5}, \frac{8}{14}$

$\frac{2}{3.5} \stackrel{?}{=} \frac{8}{14}$ Original proportion

$2(14) \stackrel{?}{=} 3.5(8)$ Cross products

$28 = 28$ ✓ Simplify.

The cross products are equal, so the ratios form a proportion.

b. $\frac{0.3}{1.5}, \frac{0.5}{2.0}$

$\frac{0.3}{1.5} \stackrel{?}{=} \frac{0.5}{2.0}$ Original proportion

$0.3(2.0) \stackrel{?}{=} 1.5(0.5)$ Cross products

$0.6 \neq 0.75$ ✗ Simplify.

The cross products are not equal, so the ratios do not form a proportion.

GuidedPractice

2A. $\frac{0.2}{1.8}, \frac{1}{0.9}$

2B. $\frac{15}{36}, \frac{35}{42}$

StudyTip

Cross Products When you find cross products, you are said to be *cross multiplying*.

2 Solve Proportions

To solve proportions, use cross products.

Example 3 Solve a Proportion

Solve each proportion. If necessary, round to the nearest hundredth.

a. $\frac{x}{10} = \frac{3}{5}$

$\frac{x}{10} = \frac{3}{5}$	Original proportion
$x(5) = 10(3)$	Find the cross products.
$5x = 30$	Simplify.
$\frac{5x}{5} = \frac{30}{5}$	Divide each side by 5.
$x = 6$	Simplify.

b. $\frac{x-2}{14} = \frac{2}{7}$

$\frac{x-2}{14} = \frac{2}{7}$	Original proportion
$(x-2)7 = 14(2)$	Find the cross products.
$7x - 14 = 28$	Simplify.
$7x = 42$	Add 14 to each side.
$x = 6$	Divide each side by 7.

GuidedPractice

3A. $\frac{r}{8} = \frac{25}{40}$

3B. $\frac{x+4}{5} = \frac{3}{8}$

The ratio of two measurements having different units of measure is called a **rate**. For example, a price of $9.99 per 10 songs is a rate. A rate that tells how many of one item is being compared to 1 of another item is called a **unit rate**.

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Real-WorldCareer

Retail Buyer A retail buyer purchases goods for stores, primarily from wholesalers, for resale to the general public. Buyers use math to determine the amount of each product to order. A bachelor's degree with an emphasis on business studies is usually required.

Real-World Example 4 Rate of Growth

RETAIL In the past two years, a retailer has opened 232 stores. If the rate of growth remains constant, how many stores will the retailer open in the next 3 years?

Understand Let r represent the number of retail stores.

Plan Write a proportion for the problem.

$$\frac{232 \text{ retail stores}}{2 \text{ years}} = \frac{r \text{ retail stores}}{3 \text{ years}}$$

Solve

$\frac{232}{2} = \frac{r}{3}$	Original proportion
$232(3) = 2r$	Find the cross products.
$696 = 2r$	Simplify.
$\frac{696}{2} = \frac{2r}{2}$	Divide each side by 2.
$348 = r$	Simplify.

The retailer will open 348 stores in 3 years.

Check If the clothing retailer continues to open 232 stores every 2 years, then in the next 3 years, it will open 348 stores.

Guided Practice

4. **EXERCISE** It takes 7 minutes for Isabella to walk around the gym track twice. At this rate, how many times can she walk around the track in a half hour?

A rate called a **scale** is used to make a **scale model** of something too large or too small to be convenient at actual size.

Real-World Link

The Great Smoky Mountains National Park in Tennessee is home to several waterfalls. The Ramsey Cascades is 100 feet tall. It is the tallest in the park.

Source: National Park Service

Real-World Example 5 Scale and Scale Models

MOUNTAIN TRAIL The Ramsey Cascades Trail is about $1\frac{1}{8}$ inches long on a map with scale 3 inches = 10 miles. What is the actual length of the trail?

Let ℓ represent the actual length.

scale → $\frac{3}{10} = \frac{1\frac{1}{8}}{\ell}$ ← scale
actual → ← actual

$3(\ell) = 1\frac{1}{8}(10)$ Find the cross products.

$3\ell = \frac{45}{4}$ Simplify.

$3\ell \div 3 = \frac{45}{4} \div 3$ Divide each side by 3.

$\ell = \frac{15}{4}$ or $3\frac{3}{4}$ Simplify.

The actual length is about $3\frac{3}{4}$ miles.

Guided Practice

5. **AIRPLANES** On a model airplane, the scale is 5 centimeters = 2 meters. If the model's wingspan is 28.5 centimeters, what is the actual wingspan?

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Examples 1–2 **Determine whether each pair of ratios are equivalent ratios. Write *yes* or *no*.**

1. $\frac{3}{7}, \frac{9}{14}$
2. $\frac{7}{8}, \frac{42}{48}$
3. $\frac{2.8}{4.4}, \frac{1.4}{2.1}$

Example 3 **Solve each proportion. If necessary, round to the nearest hundredth.**

4. $\frac{n}{9} = \frac{6}{27}$
5. $\frac{4}{u} = \frac{28}{35}$
6. $\frac{3}{8} = \frac{b}{10}$

Example 4 7. **RACE** Jennie ran the first 6 miles of a marathon in 58 minutes. If she is able to maintain the same pace, how long will it take her to finish the 26.2 miles?

Example 5 8. **CCSS PRECISION** On a map of North Carolina, Raleigh and Asheville are about 8 inches apart. If the scale is 1 inch = 12 miles, how far apart are the cities?

Jerry Whaley/Photographer's Choice/Getty Images

Practice and Problem Solving

Extra Practice is on page R2.

Examples 1–2 **Determine whether each pair of ratios are equivalent ratios. Write *yes* or *no*.**

9. $\frac{9}{11}, \frac{81}{99}$ **10.** $\frac{3}{7}, \frac{18}{42}$ **11.** $\frac{8.4}{9.2}, \frac{8.8}{9.6}$

12. $\frac{4}{3}, \frac{6}{8}$ **13.** $\frac{29.2}{10.4}, \frac{7.3}{2.6}$ **14.** $\frac{39.68}{60.14}, \frac{6.4}{9.7}$

Example 3 **Solve each proportion. If necessary, round to the nearest hundredth.**

15. $\frac{3}{8} = \frac{15}{a}$ **16.** $\frac{t}{2} = \frac{6}{12}$ **17.** $\frac{4}{9} = \frac{13}{q}$

18. $\frac{15}{35} = \frac{8}{7}$ **19.** $\frac{7}{10} = \frac{m}{14}$ **20.** $\frac{8}{13} = \frac{v}{21}$

21. $\frac{w}{2} = \frac{4.5}{6.8}$ **22.** $\frac{1}{0.19} = \frac{12}{n}$ **23.** $\frac{2}{0.21} = \frac{8}{n}$

24. $\frac{2.4}{3.6} = \frac{k}{1.8}$ **25** $\frac{t}{0.3} = \frac{1.7}{0.9}$ **26.** $\frac{7}{1.066} = \frac{z}{9.65}$

27. $\frac{x-3}{5} = \frac{6}{10}$ **28.** $\frac{7}{x+9} = \frac{21}{36}$ **29.** $\frac{10}{15} = \frac{4}{x-5}$

Example 4 **30. CAR WASH** The B-Clean Car Wash washed 128 cars in 3 hours. At that rate, how many cars can they wash in 8 hours?

Example 5 **31. GEOGRAPHY** On a map of Florida, the distance between Jacksonville and Tallahassee is 2.6 centimeters. If 2 centimeters = 120 miles, what is the distance between the two cities?

32. CCSS PRECISION An artist used interlocking building blocks to build a scale model of Kennedy Space Center, Florida. In the model, 1 inch equals 1.67 feet of an actual space shuttle. The model is 110.3 inches tall. How tall is the actual space shuttle? Round to the nearest tenth.

33. MENU On Monday, a restaurant made $545 from selling 110 hamburgers. If they sold 53 hamburgers on Tuesday, how much did they make?

Solve each proportion. If necessary, round to the nearest hundredth.

34. $\frac{6}{14} = \frac{7}{x-3}$ **35.** $\frac{7}{4} = \frac{f-4}{8}$ **36.** $\frac{3-y}{4} = \frac{1}{9}$

37. $\frac{4v+7}{15} = \frac{6v+2}{10}$ **38.** $\frac{9b-3}{9} = \frac{5b+5}{3}$ **39.** $\frac{2n-4}{5} = \frac{3n+3}{10}$

40. ATHLETES At Piedmont High School, 3 out of every 8 students are athletes. If there are 1280 students at the school, how many are not athletes?

41. BRACES Two out of five students in the ninth grade have braces. If there are 325 students in the ninth grade, how many have braces?

42. PAINT Joel used a half gallon of paint to cover 84 square feet of wall. He has 932 square feet of wall to paint. How many gallons of paint should he purchase?

43 **MOVIE THEATERS** Use the table at the right.

a. Write a ratio of the number of indoor theaters to the total number of theaters for each year.

b. Do any two of the ratios you wrote for part **a** form a proportion? If so, explain the real-world meaning of the proportion.

Year	Indoor	Drive-In	Total
2003	35,361	634	35,995
2004	36,012	640	36,652
2005	37,092	648	37,740
2006	37,776	649	38,425
2007	38,159	635	38,794
2008	38,201	633	38,834
2009	38,605	628	39,233

Source: North American Theater Owners

44. **DIARIES** In a survey, 36% of the students said that they kept an electronic diary. There were 900 students who kept an electronic diary. How many students were in the survey?

45. **MULTIPLE REPRESENTATIONS** In this problem, you will explore how changing the lengths of the sides of a shape by a factor changes the perimeter of that shape.

a. **Geometric** Draw a square *ABCD*. Draw a square *MNPQ* with sides twice as long as *ABCD*. Draw a square *FGHJ* with sides half as long as *ABCD*.

b. **Tabular** Complete the table below using the appropriate measures.

ABCD		*MNPQ*		*FGHJ*	
Side length		Side length		Side length	
Perimeter		Perimeter		Perimeter	

c. **Verbal** Make a conjecture about the change in the perimeter of a square if the side length is increased or decreased by a factor.

H.O.T. Problems Use Higher-Order Thinking Skills

46. CCSS **STRUCTURE** In 2007, organic farms occupied 2.6 million acres in the United States and produced goods worth about $1.7 billion. Divide one of these numbers by the other and explain the meaning of the result.

47. **REASONING** Compare and contrast ratios and rates.

48. **CHALLENGE** If $\frac{a+1}{b-1} = \frac{5}{1}$ and $\frac{a-1}{b+1} = \frac{1}{1}$, find the value of $\frac{b}{a}$. (*Hint:* Choose values of a and b for which the proportions are true and evaluate $\frac{b}{a}$.)

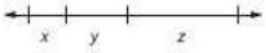
49. **WRITING IN MATH** On a road trip, Marcus reads a highway sign and then looks at his gas gauge.

Marcus's gas tank holds 10 gallons and his car gets 32 miles per gallon at his current speed of 65 miles per hour. If he maintains this speed, will he make it to Atlanta without having to stop and get gas? Explain your reasoning.

50. **WRITING IN MATH** Describe how businesses can use ratios. Write about a real-world situation in which a business would use a ratio.

Standardized Test Practice

51. In the figure, $x : y = 2 : 3$ and $y : z = 3 : 5$. If $x = 10$, find the value of z.



A 15
B 20
C 25
D 30

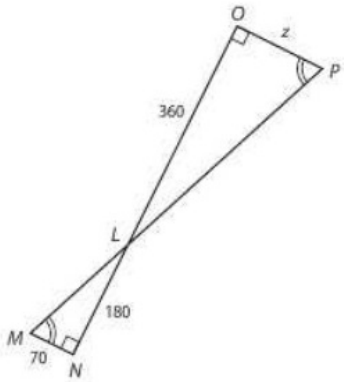
52. GRIDDED RESPONSE A race car driver records the finishing times for recent practice trials.

Trial	Time (seconds)
1	5.09
2	5.10
3	4.95
4	4.91
5	5.05

What is the mean time, in seconds, for the trials?

53. GEOMETRY If $\triangle LMN$ is similar to $\triangle LPO$, what is z?

F 240
G 140
H 120
J 70



54. Which equation below illustrates the Commutative Property?

A $(3x + 4y) + 2z = 3x + (4y + 2z)$
B $7(x + y) = 7x + 7y$
C $xyz = yxz$
D $x + 0 = x$

Spiral Review

Solve each equation. (Lesson 2-5)

55. $|x + 5| = -8$

56. $|b + 9| = 2$

57. $|2p - 3| = 17$

58. $|5c - 8| = 12$

59. HEALTH When exercising, a person's pulse rate should not exceed a certain limit. This maximum rate is represented by the expression $0.8(220 - a)$, where a is age in years. Find the age of a person whose maximum pulse rate is 122 more than their age. (Lesson 2-4)

Solve each equation. Check your solution. (Lesson 2-3)

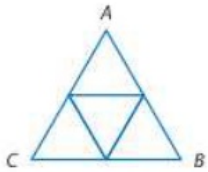
60. $15 = 4a - 5$

61. $7g - 14 = -63$

62. $9 + \frac{y}{5} = 6$

63. $\frac{t}{8} - 6 = -12$

64. GEOMETRY Find the area of $\triangle ABC$ if each small triangle has a base of 5.2 inches and a height of 4.5 inches. (Lesson 1-4)



Evaluate each expression. (Lesson 1-2)

65. $3 + 16 \div 8 \cdot 5$

66. $4^2 \cdot 3 - 5(6 + 3)$

Skills Review

Solve each equation.

67. $4p = 22$

68. $5h = 33$

69. $1.25y = 4.375$

70. $9.8m = 30.87$

EXTEND

2-6 Spreadsheet Lab
Descriptive Modeling

When using numbers to model a real-world situation, it is often helpful to have a metric. A **metric** is a rule for assigning a number to some characteristic or attribute. For example, teachers use metrics to determine grades. Each teacher determines an appropriate metric for assessing a student's performance and assigning a grade.

Common Core State Standards
Content Standards
N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.

You can use a spreadsheet to calculate different metrics.

Activity

Dorrie wants to buy a house. She has the following expenses: rent of $650, credit card monthly bills of $320, a car payment of $410, and a student loan payment of $115. Dorrie has a yearly salary of $46,500. Use a spreadsheet to find Dorrie's debt-to-income ratio.

Step 1 Enter Dorrie's debts in column B.

Step 2 Add her debts using a function in cell B6. Go to Insert and then Function. Then choose Sum. The sum of 1495 appears in B6.

Step 3 Now insert Dorrie's salary in column C. Remember to find her monthly salary by dividing the yearly salary by 12.

A mortgage company will use the debt-to-income ratio as a metric to determine if Dorrie qualifies for a loan. The **debt-to-income ratio** is calculated as *how much she owes per month* divided by *how much she earns each month*.

Step 4 Enter a formula to find the debt-to-income ratio in cell C6. In the formula bar, enter =B6/C2.

The ratio of about 0.39 appears. An ideal ratio would be 0.36 or less. A ratio higher than 0.36 would cause an increased interest rate or may require a higher down payment.

The spreadsheet shows a debt-to-income ratio of about 0.39. Dorrie should try to eliminate or reduce some debts or try to earn more money in order to lower her debt-to-income ratio.

Lab 2-6 B Spreadsheet.xls

	A	B	C
1	Type of Debt	Expenses	Salary
2	Rent	650	3875
3	Credit Cards	320	
4	Car Payment	410	
5	Student Loan	115	
6		1495	0.385806
7			

Sheet 1 Sheet 2 Sheet 3

Exercises

1. How could Dorrie improve her debt-to-income ratio?
2. Another metric mortgage companies use is the ratio of monthly mortgage to total monthly income. An ideal ratio is 0.28. Using this metric, how much could Dorrie afford to pay for a mortgage each month?
3. How effective are each of these metrics as measures of whether Dorrie can afford to buy a house? Explain your reasoning.
4. **MODELING** Metrics are used to compare athletes. For example, ERAs are used to compare pitchers. Find a metric and evaluate its effectiveness for modeling. Compare it to other metrics, and then define your own metric.

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