LESSON 4-3

Writing Equations in Point-Slope Form

Then

- You wrote linear equations given either one point and the slope or two points.

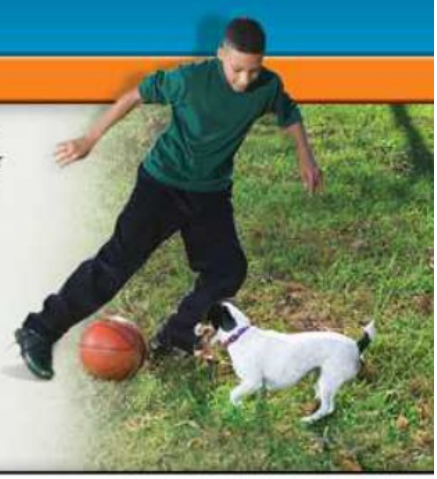
Now

- 1 Write equations of lines in point-slope form.
- 2 Write linear equations in different forms.

Why?

- Most humane societies have foster homes for newborn puppies, kittens, and injured or ill animals. During the spring and summer, a large shelter can place 3000 animals in homes each month.

If a shelter had 200 animals in foster homes at the beginning of spring, the number of animals in foster homes at the end of the summer could be represented by $y = 3000x + 200$, where x is the number of months and y is the number of animals.



New Vocabulary

point-slope form

Common Core State Standards

Content Standards

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Mathematical Practices

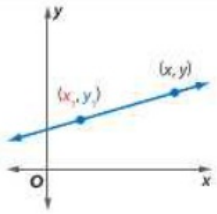
2 Reason abstractly and quantitatively.

1 Point-Slope Form

An equation of a line can be written in **point-slope form** when given the coordinates of one known point on a line and the slope of that line.

KeyConcept Point-Slope Form

Words	The linear equation $y - y_1 = m(x - x_1)$ is written in point-slope form, where (x_1, y_1) is a given point on a nonvertical line and m is the slope of the line.
Symbols	$y - y_1 = m(x - x_1)$



Example 1 Write and Graph an Equation in Point-Slope Form

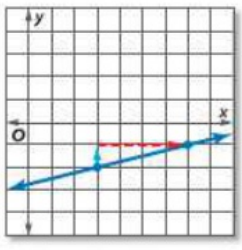
Write an equation in point-slope form for the line that passes through $(3, -2)$ with a slope of $\frac{1}{4}$. Then graph the equation.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - (-2) = \frac{1}{4}(x - 3)$ $(x_1, y_1) = (3, -2), m = \frac{1}{4}$

$y + 2 = \frac{1}{4}(x - 3)$ Simplify.

Plot the point at $(3, -2)$ and use the slope to find another point on the line. Draw a line through the two points.



Guided Practice

1. Write an equation in point-slope form for the line that passes through $(-2, 1)$ with a slope of -6. Then graph the equation.

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2 Forms of Linear Equations If you are given the slope and the coordinates of one or two points, you can write the linear equation in the following ways.

StudyTip
Slope The slope of the line remains unchanged throughout the line. You can go in either direction along the line using the same rise over run and you will always end at a point on the line.

ConceptSummary Writing Equations

Given the Slope and One Point

Step 1 Substitute the value of m and let the x and y coordinates be (x_1, y_1). Or, substitute the value of m, x, and y into the slope-intercept form and solve for b.

Step 2 Rewrite the equation in the needed form.

Given Two Points

Step 1 Find the slope.

Step 2 Choose one of the two points to use.

Step 3 Follow the steps for writing an equation given the slope and one point.

ReviewVocabulary
Standard form of a linear equation $Ax + By = C$, where $A \geq 0$, A and B are not both zero, and A, B, and C are integers with a greatest common factor of 1

Example 2 Standard Form

Write $y - 1 = -\frac{2}{3}(x - 5)$ in standard form.

$y - 1 = -\frac{2}{3}(x - 5)$	Original equation
$3(y - 1) = 3\left(-\frac{2}{3}\right)(x - 5)$	Multiply each side by 3 to eliminate the fraction.
$3(y - 1) = -2(x - 5)$	Simplify.
$3y - 3 = -2x + 10$	Distributive Property
$3y = -2x + 13$	Add 3 to each side.
$2x + 3y = 13$	Add $2x$ to each side.

GuidedPractice

2. Write $y - 1 = 7(x + 5)$ in standard form.

To find the y-intercept of an equation, rewrite the equation in slope-intercept form.

Example 3 Slope-Intercept Form

Write $y + 3 = \frac{3}{2}(x + 1)$ in slope-intercept form.

$y + 3 = \frac{3}{2}(x + 1)$	Original equation
$y + 3 = \frac{3}{2}x + \frac{3}{2}$	Distributive Property
$y = \frac{3}{2}x - \frac{3}{2}$	Subtract 3 from each side.

GuidedPractice

3. Write $y + 6 = -3(x - 4)$ in slope-intercept form.

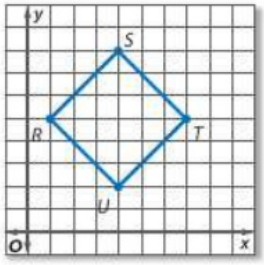
Being able to use a variety of forms of linear equations can be useful in other subjects as well.

StudyTip

Slopes in Squares
Nonvertical opposite sides of a square have equal slopes. If the coordinates for one of the vertices are unavailable, use the slope of the opposite side.

Example 4 Point-Slope Form and Standard Form

GEOMETRY **The figure shows square *RSTU*.**



a. Write an equation in point-slope form for the line containing side $\overline{TU}$.

Step 1 Find the slope of $\overline{TU}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope Formula

$$= \frac{5 - 2}{7 - 4} \text{ or } 1$$ $(x_1, y_1) = (4, 2)$ and $(x_2, y_2) = (7, 5)$

Step 2 You can select either point for (x_1, y_1) in the point-slope form.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 2 = 1(x - 4)$ $(x_1, y_1) = (4, 2)$

$y - 5 = 1(x - 7)$ $(x_1, y_1) = (7, 5)$

b. Write an equation in standard form for the same line.

$y - 2 = 1(x - 4)$	Original equation	$y - 5 = 1(x - 7)$
$y - 2 = 1x - 4$	Distributive Property	$y - 5 = 1x - 7$
$y = 1x - 2$	Add to each side.	$y = 1x - 2$
$-1x + y = -2$	Subtract $1x$ from each side.	$-1x + y = -2$
$x - y = 2$	Multiply each side by -1.	$x - y = 2$

GuidedPractice

4A. Write an equation in point-slope form of the line containing side $\overline{ST}$.

4B. Write an equation in standard form of the line containing $\overline{ST}$.

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1 **Write an equation in point-slope form for the line that passes through the given point with the slope provided. Then graph the equation.**

1. $(-2, 5)$, slope -6
2. $(-2, -8)$, slope $\frac{5}{6}$
3. $(4, 3)$, slope $-\frac{1}{2}$

Example 2 **Write each equation in standard form.**

4. $y + 2 = \frac{7}{8}(x - 3)$
5. $y + 7 = -5(x + 3)$
6. $y + 2 = \frac{5}{3}(x + 6)$

Example 3 **Write each equation in slope-intercept form.**

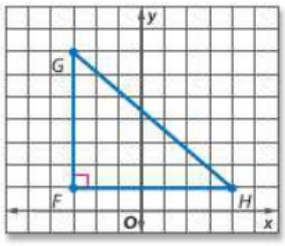
7. $y - 10 = 4(x + 6)$
8. $y - 7 = -\frac{3}{4}(x + 5)$
9. $y - 9 = x + 4$

Example 4

10. GEOMETRY Use right triangle FGH.

a. Write an equation in point-slope form for the line containing $\overline{GH}$.

b. Write the standard form of the line containing $\overline{GH}$.



Practice and Problem Solving

Extra Practice is on page R4.

Example 1

Write an equation in point-slope form for the line that passes through each point with the given slope. Then graph the equation.

11. $(5, 3)$, $m = 7$

12. $(2, -1)$, $m = -3$

13. $(-6, -3)$, $m = -1$

14. $(-7, 6)$, $m = 0$

15. $(-2, 11)$, $m = \frac{4}{3}$

16. $(-6, -8)$, $m = -\frac{5}{8}$

17. $(-2, -9)$, $m = -\frac{7}{5}$

18. $(-6, 0)$, horizontal line

Example 2

Write each equation in standard form.

19. $y - 10 = 2(x - 8)$

20. $y - 6 = -3(x + 2)$

21. $y - 9 = -6(x + 9)$

22. $y + 4 = \frac{2}{3}(x + 7)$

23. $y + 7 = \frac{9}{10}(x + 3)$

24. $y + 7 = -\frac{3}{2}(x + 1)$

25. $2y + 3 = -\frac{1}{3}(x - 2)$

26. $4y - 5x = 3(4x - 2y + 1)$

Example 3

Write each equation in slope-intercept form.

27. $y - 6 = -2(x - 7)$

28. $y - 11 = 3(x + 4)$

29. $y + 5 = -6(x + 7)$

30. $y - 1 = \frac{4}{5}(x + 5)$

31. $y + 2 = \frac{1}{6}(x - 4)$

32. $y + 6 = -\frac{3}{4}(x + 8)$

33. $y + 3 = -\frac{1}{3}(2x + 6)$

34. $y + 4 = 3(3x + 3)$

Example 4

35. MOVIE RENTALS The number of copies of a movie rented at a video kiosk decreased at a constant rate of 5 copies per week. The 6th week after the movie was released, 4 copies were rented. How many copies were rented during the second week?

36. CCSS REASONING A company offers premium cable for \$39.95 per month plus a one-time setup fee. The total cost for setup and 6 months of service is \$264.70.

a. Write an equation in point-slope form to find the total price y for any number of months x. (*Hint*: The point (6, 264.70) is a solution to the equation.)

b. Write the equation in slope-intercept form.

c. What is the setup fee?

Write an equation for the line described in standard form.

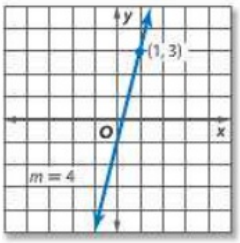
37. through $(-1, 7)$ and $(8, -2)$

38. through $(-4, 3)$ with y-intercept 0

39. with x-intercept 4 and y-intercept 5

Write an equation in point-slope form for each line.

40.



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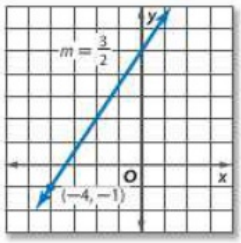

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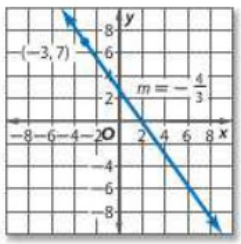


Write each equation in slope-intercept form.

43 $y + \frac{3}{5} = x - \frac{2}{5}$

44. $y - \frac{7}{2} = \frac{1}{2}(x - 4)$

45. $y + \frac{1}{3} = \frac{5}{6}\left(x + \frac{2}{5}\right)$

46. Write an equation in point-slope form, slope-intercept form, and standard form for a line that passes through $(-2, 8)$ with slope $\frac{8}{5}$.

47. Line ℓ passes through $(-9, 4)$ with slope $\frac{4}{7}$. Write an equation in point-slope form, slope-intercept form, and standard form for line ℓ.

48. WEATHER The barometric pressure is 598 millimeters of mercury (mmHg) at an altitude of 1.8 kilometers and 577 millimeters of mercury at 2.1 kilometers.

a. Write a formula for the barometric pressure as a function of the altitude.

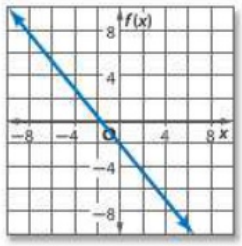
b. What is the altitude if the pressure is 657 millimeters of mercury?

H.O.T. Problems Use Higher-Order Thinking Skills

49. WHICH ONE DOESN'T BELONG? Identify the equation that does not belong. Explain your reasoning.

$y - 5 = 3(x - 1)$	$y + 1 = 3(x + 1)$	$y + 4 = 3(x + 1)$	$y - 8 = 3(x - 2)$

50. CCSS CRITIQUE Juana thinks that $f(x)$ and $g(x)$ have the same slope but different intercepts. Sabrina thinks that $f(x)$ and $g(x)$ describe the same line. Is either of them correct? Explain your reasoning.



The graph of $g(x)$ is the line that passes through $(3, -7)$ and $(-6, 4)$.

51. OPEN ENDED Describe a real-life scenario that has a constant rate of change and a value of y for a particular value of x. Represent this situation using an equation in point-slope form, an equation in standard form, and an equation in slope-intercept form.

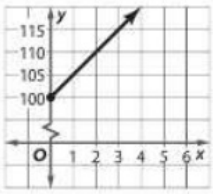
52. REASONING Write an equation for the line that passes through $(-4, 8)$ and $(3, -7)$. What is the slope? Where does the line intersect the x-axis? the y-axis?

53. CHALLENGE Write an equation in point-slope form for the line that passes through the points (f, g) and (h, j).

54. WRITING IN MATH Why do we represent linear equations in more than one form?

Standardized Test Practice

55. Which statement is *most* strongly supported by the graph?



A You have \$100 and spend \$5 weekly.

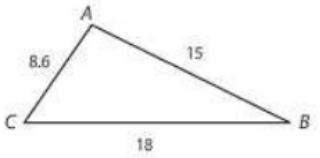
B You have \$100 and save \$5 weekly.

C You need \$100 for a new CD player and save \$5 weekly.

D You need \$100 for a new CD player and spend \$5 weekly.

56. SHORT RESPONSE A store offers customers a \$5 gift certificate for every \$75 they spend. How much would a customer have to spend to earn \$35 worth of gift certificates?

57. GEOMETRY Which triangle is similar to $\triangle ABC$?



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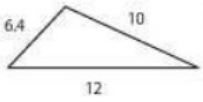

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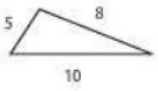

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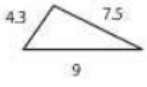

58. In a class of 25 students, 6 have blue eyes, 15 have brown hair, and 3 have blue eyes and brown hair. How many students have neither blue eyes nor brown hair?

A 4 **C** 10

B 7 **D** 22

Spiral Review

Write an equation of the line that passes through each pair of points. (Lesson 4-2)

59. (4, 2), (−2, −4) **60.** (3, −2), (6, 4) **61.** (−1, 3), (2, −3)

62. (2, −2), (3, 2) **63.** (7, −2), (−4, −2) **64.** (0, 5), (−3, 5)

Write an equation in slope-intercept form of the line with the given slope and *y*-intercept. (Lesson 4-1)

65. slope: −2, *y*-intercept: 6 **66.** slope: 3, *y*-intercept: −5 **67.** slope: $\frac{1}{2}$, *y*-intercept: 3

68. slope: $-\frac{3}{5}$, *y*-intercept: 12 **69.** slope: 0, *y*-intercept: 3 **70.** slope: −1, *y*-intercept: 0

71. THEATER The Coral Gables Actors' Playhouse has 7 rows of seats in the orchestra section. The number of seats in the rows forms an arithmetic sequence, as shown in the table. On opening night, 368 tickets were sold for the orchestra section. Was the section oversold? (Lesson 3-5)

Rows	Number of Seats
7	76
6	68
5	60

Skills Review

Solve each equation or formula for the variable specified.

72. $y = mx + b$, for m **73.** $v = r + at$, for a

74. $km + 5x = 6y$, for m **75.** $4b - 5 = -t$, for b