LESSON 5-3

Solving Multi-Step Inequalities

Then	Now	Why?
• You solved multi-step equations.	• 1 Solve linear inequalities involving more than one operation. 2 Solve linear inequalities involving the Distributive Property.	• A salesperson may make a base monthly salary and earn a commission on each of her sales. To find the number of sales she needs to make to pay her monthly bills, you can use a multi-step inequality.

Common Core State Standards

Content Standards

A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematical Practices

7 Look for and make use of structure.

1 Solve Multi-Step Inequalities

Multi–step inequalities can be solved by undoing the operations in the same way you would solve a multi-step equation.

Real-World Example 1 Solve a Multi-Step Inequality

SALES **Write and solve an inequality to find the sales Mrs. Jones needs if she earns a monthly salary of $2000 plus a 10% commission on her sales. Her goal is to make at least $4000 per month. What sales does she need to meet her goal?**

base salary + (commission × sales) ≥ income needed

$2000 + 0.10x \geq 4000$ Substitution

$0.10x \geq 2000$ Subtract 2000 from each side.

$x \geq 20{,}000$ Divide each side by 0.10.

She must make at least $20,000 in sales to meet her monthly goal.

Guided Practice

1. **FINANCIAL LITERACY** The Print Shop advertises a special to print 400 flyers for less than the competition. The price includes a $3.50 set-up fee. If the competition charges $35.50, what does the Print Shop charge for each flyer?

When multiplying or dividing by a negative number, the direction of the inequality symbol changes. This holds true for multi-step inequalities.

Example 2 Inequality Involving a Negative Coefficient

Solve $-11y - 13 > 42$. Graph the solution on a number line.

$-11y - 13 > 42$ Original Inequality

$-11y > 55$ Add 13 to each side and simplify.

$\frac{-11y}{-11} < \frac{55}{-11}$ Divide each side by −11, and reverse the inequality.

$y < -5$ Simplify.

The solution set is $\{y \mid y < -5\}$.

Guided Practice **Solve each inequality.**

2A. $23 \geq 10 - 2w$

2B. $43 > -4y + 11$

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You can translate sentences into multi-step inequalities and then solve them using the Properties of Inequalities.

Real-World Career

Veterinarian Veterinarians take care of sick and injured animals. Vets can work anywhere from a zoo to a research facility to owning their own practice. Vets need to earn a bachelor's degree, attend vet college for 4 years, and take a test to get licensed.

Example 3 Write and Solve an Inequality

Define a variable, write an inequality, and solve the problem.

Five minus 6 times a number is more than four times the number plus 45.

Let n be the number.

Five	minus	six times a number	is more than	four times a number	plus	forty-five.
5	$-$	$6n$	$>$	$4n$	$+$	45

$5 - 10n > 45$	Subtract $4n$ from each side and simplify.
$-10n > 40$	Subtract 5 from each side and simplify.
$\frac{-10n}{-10} < \frac{40}{-10}$	Divide each side by -10, and reverse the inequality.
$n < -4$	Simplify.

The solution set is $\{n \mid n < -4\}$.

Guided Practice

3. *Two more than half of a number is greater than twenty-seven.*

2 Solve Inequalities Involving the Distributive Property

When solving inequalities that contain grouping symbols, use the Distributive Property to remove the grouping symbols first. Then use the order of operations to simplify the resulting inequality.

Example 4 Distributive Property

Solve $4(3t - 5) + 7 \geq 8t + 3$. Graph the solution on a number line.

$4(3t - 5) + 7 \geq 8t + 3$	Original inequality
$12t - 20 + 7 \geq 8t + 3$	Distributive Property
$12t - 13 \geq 8t + 3$	Combine like terms.
$4t - 13 \geq 3$	Subtract $8t$ from each side and simplify.
$4t \geq 16$	Add 13 to each side.
$\frac{4t}{4} \geq \frac{16}{4}$	Divide each side by 4.
$t \geq 4$	Simplify.

The solution set is $\{t \mid t \geq 4\}$.

Guided Practice

Solve each inequality. Graph the solution on a number line.

4A. $6(5z - 3) \leq 36z$

4B. $2(h + 6) > -3(8 - h)$

Watch Out!

Distributive Property

If a negative number is multiplied by a sum or difference, remember to distribute the negative sign along with the number to each term inside the parentheses.

If solving an inequality results in a statement that is always true, the solution set is the set of all real numbers. This solution set is written as $\{x \mid x \text{ is a real number.}\}$. If solving an inequality results in a statement that is never true, the solution set is the empty set, which is written as the symbol $\varnothing$. The empty set has no members.

Masterfile

Example 5 Empty Set and All Reals

Solve each inequality. Check your solution.

a. $9t - 5(t - 5) \leq 4(t - 3)$

$9t - 5(t - 5) \leq 4(t - 3)$	Original inequality
$9t - 5t + 25 \leq 4t - 12$	Distributive Property
$4t + 25 \leq 4t - 12$	Combine like terms.
$4t + 25 - 4t \leq 4t - 12 - 4t$	Subtract $4t$ from each side.
$25 \leq -12$	Simplify.

Since the inequality results in a false statement, the solution set is the empty set, $\varnothing$.

b. $3(4m + 6) \leq 42 + 6(2m - 4)$

$3(4m + 6) \leq 42 + 6(2m - 4)$	Original inequality
$12m + 18 \leq 42 + 12m - 24$	Distributive Property
$12m + 18 \leq 12m + 18$	Combine like terms.
$12m + 18 - 12m \leq 12m + 18 - 12m$	Subtract $12m$ from each side.
$18 \leq 18$	Simplify.

All values of m make the inequality true. All real numbers are solutions.

StudyTip

CCSS Structure Notice that the inequality $4t + 25 < 4t - 12$ means *some number 4t plus 25 is less than or equal to that number minus 12.* No real number makes that inequality true. Observing the meaning of the expressions in each step in this way can lead you to solutions more quickly.

GuidedPractice

Solve each inequality. Check your solution.

5A. $18 - 3(8c + 4) \geq -6(4c - 1)$ **5B.** $46 \leq 8m - 4(2m + 5)$

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1

1. CANOEING If four people plan to use the canoe with 60 pounds of supplies, write and solve an inequality to find the allowable average weight per person.

800-lb capacity

2. SHOPPING Rita is ordering a movie for \$11.95 and a few CDs. She has \$50 to spend. Shipping and sales tax will be \$10. If each CD costs \$9.99, write and solve an inequality to find the greatest number of CDs that she can buy.

Example 2

CCSS **STRUCTURE** **Solve each inequality. Graph the solution on a number line.**

3. $6h - 10 \geq 32$ **4.** $-3 \leq \frac{2}{3}r + 9$

5. $-3x + 7 > 43$ **6.** $4m - 17 < 6m + 25$

Example 3

Define a variable, write an inequality, and solve each problem. Then check your solution.

7. Four times a number minus 6 is greater than eight plus two times the number.

8. Negative three times a number plus 4 is less than five times the number plus 8.

Examples 4–5 **Solve each inequality. Graph the solution on a number line.**

9. $-6 \leq 3(5v - 2)$ **10.** $-5(g + 4) > 3(g - 4)$ **11.** $3 - 8x \geq 9 + 2(1 - 4x)$

Practice and Problem Solving

Extra Practice is on page R5.

Examples 1 and 2

CCSS **STRUCTURE** **Solve each inequality. Graph the solution on a number line.**

12. $5b - 1 \geq -11$

13. $21 > 15 + 2a$

14. $-9 \geq \frac{2}{5}m + 7$

15. $\frac{w}{8} - 13 > -6$

16. $-a + 6 \leq 5$

17. $37 < 7 - 10w$

18. $8 - \frac{z}{3} \geq 11$

19. $-\frac{5}{4}p + 6 < 12$

20. $3b - 6 \geq 15 + 24b$

21. $15h + 30 < 10h - 45$

Example 3

Define a variable, write an inequality, and solve each problem. Check your solution.

22. Three fourths of a number decreased by nine is at least forty-two.

23. Two thirds of a number added to six is at least twenty-two.

24. Seven tenths of a number plus 14 is less than forty-nine.

25. Eight times a number minus twenty-seven is no more than the negative of that number plus eighteen.

26. Ten is no more than 4 times the sum of twice a number and three.

27. Three times the sum of a number and seven is greater than five times the number less thirteen.

28. The sum of nine times a number and fifteen is less than or equal to the sum of twenty-four and ten times the number.

Examples 4 and 5

CCSS **STRUCTURE** **Solve each inequality. Graph the solution on a number line.**

29. $-3(7n + 3) < 6n$

30. $21 \geq 3(a - 7) + 9$

31. $2y + 4 > 2(3 + y)$

32. $3(2 - b) < 10 - 3(b - 6)$

33. $7 + t \leq 2(t + 3) + 2$

34. $8a + 2(1 - 5a) \leq 20$

Define a variable, write an inequality, and solve each problem. Then interpret your solution.

35. CARS A car salesperson is paid a base salary of \$35,000 a year plus 8% of sales. What are the sales needed to have an annual income greater than \$65,000?

36. ANIMALS Keith's dog weighs 90 pounds. A healthy weight for his dog would be less than 75 pounds. If Keith's dog can lose an average of 1.25 pounds per week on a certain diet, after how long will the dog reach healthy weight?

37. Solve $6(m - 3) > 5(2m + 4)$. Show each step and justify your work.

38. Solve $8(a - 2) \leq 10(a + 2)$. Show each step and justify your work.

39. MUSICAL A high school drama club is performing a musical to benefit a local charity. Tickets are \$5 each. They also received donations of \$565. They want to raise at least \$1500.

a. Write an inequality that describes this situation. Then solve the inequality.

b. Graph the solution.

40. ICE CREAM Benito has \$6 to spend. A sundae costs \$3.25 plus \$0.65 per topping. Write and solve an inequality to find how many toppings he can order.

41. **SCIENCE** The normal body temperature of a camel is 97.7°F in the morning. If it has had no water by noon, its body temperature can be greater than 104°F.

   a. Write an inequality that represents a camel's body temperature at noon if the camel had no water.

   b. If C represents degrees Celsius, then $F = \frac{9}{5}C + 32$. Write and solve an inequality to find the camel's body temperature at noon in degrees Celsius.

42. **NUMBER THEORY** Find all sets of three consecutive positive even integers with a sum no greater than 36.

43. **NUMBER THEORY** Find all sets of four consecutive positive odd integers with a sum that is less than 42.

Solve each inequality. Check your solution.

44. $2(x - 4) \leq 2 + 3(x - 6)$

45. $\frac{2x - 4}{6} \geq -5x + 2$

46. $5.6z + 1.5 < 2.5z - 4.7$

47. $0.7(2m - 5) \geq 21.7$

GRAPHING CALCULATOR **Use a graphing calculator to solve each inequality.**

48. $3x + 7 > 4x + 9$

49. $13x - 11 \leq 7x + 37$

50. $2(x - 3) < 3(2x + 2)$

51. $\frac{1}{2}x - 9 < 2x$

52. $2x - \frac{2}{3} \geq x - 22$

53. $\frac{1}{3}(4x + 3) \geq \frac{2}{3}x + 2$

54. **MULTIPLE REPRESENTATIONS** In this problem, you will solve compound inequalities. A number x is greater than 4, and the same number is less than 9.

   a. **Numerical** Write two separate inequalities for the statement.

   b. **Graphical** Graph the solution set for the first inequality in red. Graph the solution set for the second inequality in blue. Highlight where they overlap.

   c. **Tabular** Make a table using ten points from your number line, including points from each section. Use one column for each inequality and a third column titled "Both are True." Complete the table by writing true or false.

   d. **Verbal** Describe the relationship between the colored regions of the graph and the chart.

   e. **Logical** Make a prediction of what the graph of $4 < x < 9$ looks like.

H.O.T. Problems Use Higher-Order Thinking Skills

55. **REASONING** Explain how you could solve $-3p + 7 \geq -2$ without multiplying or dividing each side by a negative number.

56. **CHALLENGE** If $ax + b < ax + c$ is true for all real values of x, what will be the solution of $ax + b > ax + c$? Explain how you know.

57. **CHALLENGE** Solve each inequality for x. Assume that $a > 0$.

   a. $ax + 4 \geq -ax - 5$

   b. $2 - ax < x$

   c. $-\frac{2}{a}x + 3 > -9$

58. **WHICH ONE DOESN'T BELONG?** Name the inequality that does not belong. Explain.

$4y + 9 > -3$	$3y - 4 > 5$	$-2y + 1 < -5$	$-5y + 2 < -13$

59. **WRITING IN MATH** Explain when the solution set of an inequality will be the empty set or the set of all real numbers. Show an example of each.

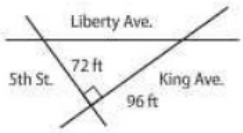
Standardized Test Practice

60. What is the solution set of the inequality $4t + 2 < 8t - (6t - 10)$?

A $\{t \mid t < -6.5\}$ C $\{t \mid t < 4\}$
B $\{t \mid t > -6.5\}$ D $\{t \mid t > 4\}$

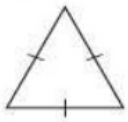
61. GEOMETRY The section of Liberty Ave. between 5th St. and King Ave. is temporarily closed. Traffic is being detoured right on 5th St., left on King Ave. and then back on Liberty Ave. How long is the closed section of Liberty Ave.?

F 100 ft
G 120 ft
H 144 ft
J 180 ft



62. SHORT RESPONSE Rhiannon is paid $52 for working 4 hours. At this rate, how many hours will it take her to earn $845?

63. GEOMETRY Classify the triangle.



A right
B parallel
C obtuse
D equilateral

Spiral Review

Solve each inequality. Check your solution. (Lesson 5-2)

64. $\frac{y}{2} \leq -5$ **65.** $12b > -48$ **66.** $-\frac{2}{3}t \leq -30$

Solve each inequality. Check your solution, and graph it on a number line. (Lesson 5-1)

67. $6 - h > -8$ **68.** $p - 9 < 2$ **69.** $3 \geq 4 - m$

Solve each equation by graphing. Verify your answer algebraically. (Lesson 3-2)

70. $2x - 7 = 4x + 9$ **71.** $5 + 3x = 7x - 11$ **72.** $2(x - 3) = 5x + 12$

73. THEME PARKS In a recent year, 70.9 million people visited the top 5 theme parks in North America. That represents an increase of about 1.14% in the number of visitors from the prior year. About how many people visited the top 5 theme parks in North America in the prior year? (Lesson 2-7)

If $f(x) = 4x - 3$ and $g(x) = 2x^2 + 5$, find each value. (Lesson 1-7)

74. $f(-2)$ **75.** $g(2) - 5$ **76.** $f(c + 3)$

77. COSMETOLOGY On average, a barber received a tip of $4 for each of 12 haircuts. Write and evaluate an expression to determine the total amount that she earned. (Lesson 1-4)

Skills Review

Graph each set of numbers on a number line.

78. $\{-4, -2, 2, 4\}$
79. $\{-3, 0, 1, 5\}$
80. {integers less than 3}
81. {integers greater than or equal to −2}
82. {integers between −3 and 4}
83. {integers less than −1}