

It's All Relative

1

Introduction to Ratio and Ratio Reasoning

WARM UP

Write a fraction to represent each situation.

1. the number of boys in your math class compared to the number of students in the class
2. the number of girls in your math class compared to the number of students in the class
3. the number of students in your math class that are absent today compared to the total number of students in the class
4. the number of students in your math class that are in attendance today compared to the total number of students in your class

LEARNING GOALS

- Distinguish between additive and multiplicative relationships between two quantities.
- Understand the concept of a ratio: a ratio represents a multiplicative comparison between two quantities.
- Write ratios in different forms and use ratio language to represent relationships between two quantities.
- Distinguish between part-to-part and part-to-whole ratios.
- Understand that fractions are part-to-whole ratios between two quantities.
- Understand that percents are part-to-whole ratios between a quantity and 100.

KEY TERMS

- additive reasoning
- multiplicative reasoning
- ratio
- percent

In elementary school, you made many comparisons using addition and subtraction. You answered questions like, "If Johnny has 9 apples and Suzie has 12 apples, who has more apples?" Is there another way to compare values?

Getting Started

Predict the Score

The Crusaders and the Blue Jays just finished the first half of their basketball game.

	Halftime Score	Final Score
Crusaders	30	?
Blue Jays	20	?

1. Predict the final score. Explain your reasoning.



Robena and Eryn each predicted the final score of a basketball game between the Crusaders and the Blue Jays.

1. Analyze each prediction.

a. Describe the reasoning that Robena and Eryn used to make each statement.

Robena

	Halftime Score	Final Score
Crusaders	30	60
Blue Jays	20	40



I think the final score will be double the score at halftime.

Eryn

	Halftime Score	Final Score
Crusaders	30	50
Blue Jays	20	40



I think the Crusaders will play hard enough to stay 10 points ahead of the Blue Jays.

b. Which team had a better second half in each prediction?

One of the students used *additive reasoning* to make her comparison and the other used *multiplicative reasoning*. **Additive reasoning** focuses on the use of addition and subtraction for comparisons. **Multiplicative reasoning** focuses on the use of multiplication and division.

c. Which student used additive reasoning and which used multiplicative reasoning?

Vicki and her nephew Benjamin share the same birthday. They were both born on March 4.

Vicki: "Today I'm 40 years old, and you're 10. I'm 4 times as old as you are!"

Benjamin: "Wow, you're old!"

Vicki: "Yeah, but in 5 years, I'll be 45, and you'll be 15. Then I will only be three times as old as you."

Benjamin: "I'm catching up to you!"

Vicki: "And 15 years after that, I'll be 60 and you'll be 30. Then I'll only be twice as old as you!"

Benjamin: "In enough time, I'll be older than you, Aunt Vicki!"



2. Is Vicki correct about how their ages change? Is Benjamin correct in thinking that he will eventually be older than his aunt?

3. The table represents the different statements from this problem situation. Let V represent Vicki's age and B represent Benjamin's age.

a. Complete the last column by identifying each relationship as either additive or multiplicative.

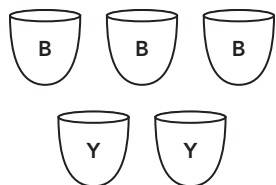
Verbal	Numeric	Relationship	
Today I'm 40 years old, and you're 10.	$V = 40, B = 10$	$V = B + 30$	
I'm 4 times as old as you are!	$V = 40, B = 10$	$V = 4B$	
Yeah, but in 5 years, I'll be 45, and you'll be 15.	$V = 45, B = 15$	$V = B + 30$	
Then I will only be three times as old as you.	$V = 45, B = 15$	$V = 3B$	
And 15 years after that, I'll be 60 and you'll be 30.	$V = 60, B = 30$	$V = B + 30$	
Then I'll only be twice as old as you!	$V = 60, B = 30$	$V = 2B$	

b. At any point in this age scenario, which relationship does not change?



The school colors at Riverview Middle School are a shade of bluish green and white. The art teacher, Mr. Raith, knows to get the correct color of bluish green it takes 3 parts blue paint to every 2 parts yellow paint.

There are different ways to think about this relationship and make comparisons. One way is to draw a picture or model.



From the model, you can make comparisons of the different quantities.

- blue parts to yellow parts
- yellow parts to blue parts
- blue parts to total parts
- yellow parts to total parts

Each comparison is called a *ratio*. A **ratio** is a comparison of two quantities that uses division. The first two comparisons are part-to-part ratios because you are comparing the individual quantities. The last two comparisons are part-to-whole ratios because you are comparing one of the parts (either blue or yellow) to the total number of parts.

Suppose Mr. Raith needs 2 parts blue paint and 5 parts yellow paint to make green paint.

1. Compare the quantities of blue and yellow paint in Mr. Raith's mixture by writing all possible ratios for each type.

a. part-to-part ratios

b. part-to-whole ratios

So you are never in doubt what a ratio represents ... label all quantities with the units of measure!



What is the difference between the part-to-part ratios that you wrote?

What is the difference between the part-to-whole ratios that you wrote?



Ratios can be found all around you, even in your classroom!
Just consider two different quantities.

For example, how many students in your class are wearing sneakers?
How many students in your class are wearing another type of shoe?

1. Use a ratio to describe the relationship given.

- a. Write a part-to-part ratio comparing the number of students wearing sneakers to the number of students wearing a different type of shoe.

- b. Write a part-to-part ratio comparing the number of students wearing a shoe other than sneakers to the number of students wearing sneakers.

- c. Write a part-to-whole ratio comparing the number of students wearing sneakers to the total number of students in the class.

- d. Write a part-to-whole ratio comparing the number of students wearing a type of shoe other than sneakers to the total number of students in the class.

Let's go on a Ratio Hunt!

2. Search around your classroom for at least 3 pairs of quantities to compare. For each pair:

- Identify the two quantities that are being compared using ratios.
- Write all possible part-to-part and/or part-to-whole comparisons of the quantities.
- Identify each ratio as part-to-part or as part-to-whole.
- Be prepared to share your treasures from the Ratio Hunt with the class.

a. Quantities being compared:

Ratio(s):

b. Quantities being compared:

Ratio(s):

c. Quantities being compared:

Ratio(s):



The Lanterton Middle School is adopting a new nickname. They have narrowed their search to the following two names: Tigers or Lions. To choose a nickname, they conducted a school-wide survey and tallied all the votes.

Each homeroom analyzed the results of the school-wide survey and reported the results in a different way.

Homeroom 6A

The votes for Tigers outnumbered the votes for Lions by a ratio of 240 to 160.

Homeroom 6B

There were 80 more votes for Tigers than Lions.

Homeroom 7A

The votes for Tigers outnumbered votes for Lions by a ratio of 3 to 2.

Homeroom 7B

3 out of 5 votes were for Tigers.

1. Describe the meaning of each statement. Then identify which describe ratios, and if so, whether the ratios are part-to-part or part-to-whole ratios.

Fractional form simply means writing the relationship in the form $\frac{a}{b}$. Just because a ratio looks like a fraction does not mean it is representing a part-to-whole comparison.

WORKED EXAMPLE

Let's consider the results reported by Homeroom 7A:
 "The votes for Tigers outnumbered votes for Lions by a ratio of 3 to 2."

This comparison is an example of a part-to-part ratio expressed in words. There are two other ways you can express this part-to-part ratio.

With a Colon

3 votes for Tigers : 2 votes for Lions

In Fractional Form

$\frac{3 \text{ votes for Tigers}}{2 \text{ votes for Lions}}$

Next, let's consider the results of the student vote as reported by Homeroom 7B: "3 out of 5 votes were for Tigers."

2. Complete the part-to-whole and part-to-part ratios written in words. Then write each ratio with a colon and in fractional form. Label all quantities.

Part-to-Whole Ratio

In Words	With a Colon	In Fractional Form
3 out of 5 votes were for Tigers.		
____ out of 5 votes were for Lions.		

Part-to-Part Ratio

In Words	With a Colon	In Fractional Form
____ votes for Tigers for every 2 votes for Lions.		
2 votes for Lions for every ____ votes for Tigers.		

Finally, let's consider the results of the survey as reported by Homeroom 6A: "The votes for Tigers outnumbered the votes for Lions by a ratio of 240 to 160.

- 3. Complete the part-to-whole and part-to-part ratios written in words. Then write each ratio with a colon and in fractional form. Label all quantities.**

Part-to-Whole Ratio

In Words	With a Colon	In Fractional Form
____ votes out of ____ votes were for Tigers.		
____ votes out of ____ votes were for Lions.		

Part-to-Part Ratio

In Words	With a Colon	In Fractional Form
____ votes for Tigers ____ votes for Lions.		
____ votes for Lions ____ votes for Tigers.		

- 4. Based on the survey, which mascot name was preferred?**



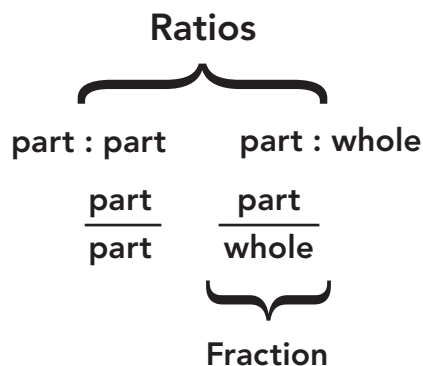
Representing a ratio in fractional form, $\frac{a}{b}$, does not have the same meaning as saying a ratio is also a fraction. This is one reason units are critical when writing ratios.

Consider each statement.

- There is an 80 percent chance of rain tomorrow.
- He ate $\frac{2}{5}$ of the cake.
- Sales tax in Greenmont is 7 percent.
- Three-fourths of the class is absent.

The situations described are examples of special types of ratios: fractions and percents.

Notice that when you write a ratio using the total number of parts, you are also writing a fraction. A fraction can be used as a ratio that shows a part-to-whole relationship.



A **percent** is a part-to-whole ratio where the whole is equal to 100. Percent is another name for hundredths. The percent symbol “%” means “per 100,” or “out of 100.” Therefore:

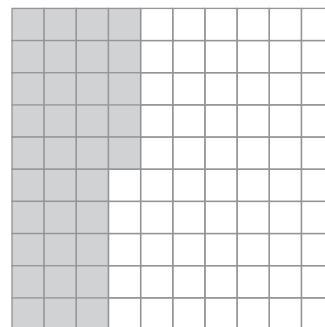
35% means 35 out of 100.

35% as a fraction is $\frac{35}{100}$.

35% as a decimal is 0.35.

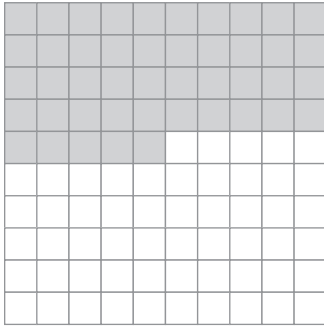
35% as a ratio is 35 to 100,
or 35 : 100.

You can shade 35 of the 100 squares on the hundredths grid to represent 35%.

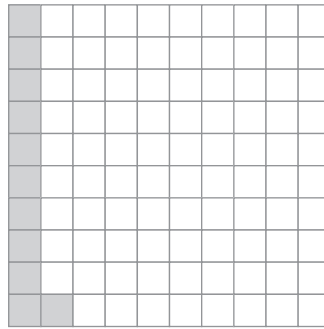


1. Each hundredths grid represents a whole. Write a fraction and a percent to represent the shaded part of each grid.

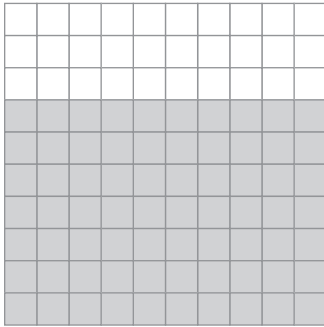
a.



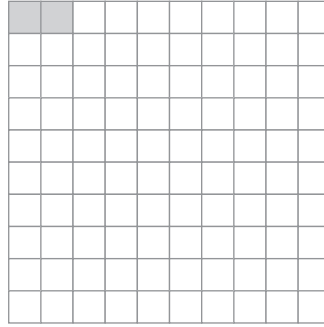
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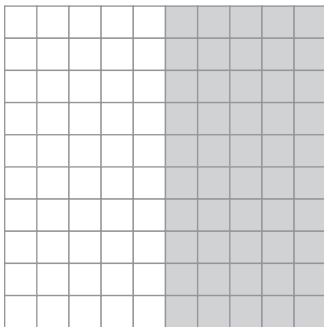
c.



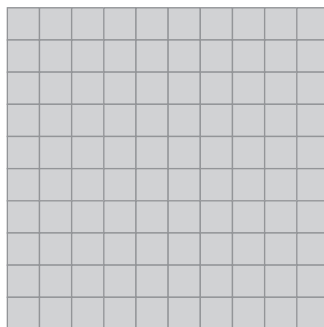
d.



e.

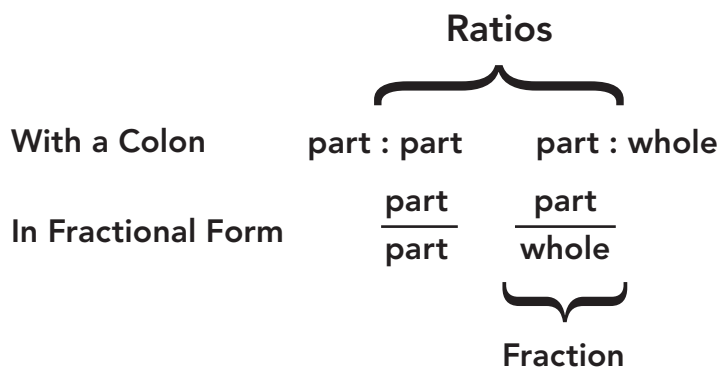


f.



TALK the TALK **Writing and Classifying Ratios**

There are several ways to compare two quantities and write ratios.



1. Consider the statement: There are s sixth grade band members and t total sixth graders.

a. Write a part-to-whole ratio using colon notation.

b. Write a part-to-part ratio using colon notation.

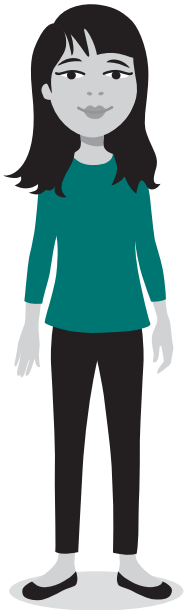
2. A survey of sixth graders with pets revealed that c students prefer cats and d students prefer dogs.
- How would you compare these two statements using part-to-part ratios?
 - How would you compare these two statements using part-to-whole ratios?
3. Analyze each statement. Determine whether a part : part or a part : whole relationship exists. Explain your reasoning.
- There are 9 girls for every 2 boys in art class.
 - Three out of every five students in art class will help paint the mural in the library.

c. There are 3 blueberry muffins to every bran muffin in a variety pack.

d. Of the 30 students in chorus, 14 of them play the piano.

e. The students planted 22 yellow daffodils and 10 white daffodils.

“
Think about
the quantities
you are
comparing.
”



Assignment

Write

Describe two ratios in the real world. Write about at least one part-to-whole ratio and one part-to-part ratio.

Remember

A ratio is a comparison of two quantities using division.

A part-to-whole ratio compares a part of a whole to the total number of parts.

A part-to-part ratio compares parts.

A part-to-whole ratio is a fraction.

A percent is a fraction in which the denominator is 100.

Practice

The Lewis brothers just joined MovieQ, a club that provides them with free movies based on a list that they pre-select. The boys work together to pick the first 10 movies for their list, each brother adding to the list based on their favorite type of movie. John David puts 5 sports movies on the list; Parker chooses 3 war movies; and Stephen adds 2 comedies.

Write the ratio in colon and in fractional form to express each relationship.

1. sports movies to war movies
2. comedies to total movies
3. war movies to comedies
4. sports movies to total movies
5. comedies to sports movies
6. war movies to total movies

Stretch

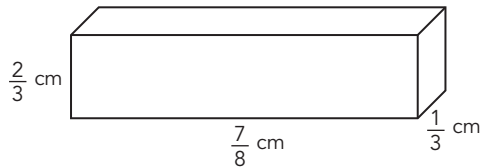
During the 2015 regular season, the Pittsburgh Pirates won 98 baseball games, 53 of which were won in their home stadium. The regular season includes 162 games.

Write a ratio for each and identify it as part-to-whole or part-to-part.

1. number of games won to number of games lost
2. number of games won to number of games played
3. number of games lost to number of games played
4. number of games won at home to number of games won away
5. number of games won at home to number of games won

Review

1. A right rectangular prism is shown.



- a. Determine the volume of the prism.
- b. Determine the surface area of the prism.
2. Estimate each sum or difference to the nearest whole number. Then calculate each sum or difference.
- a. Cristina wants to purchase four items at the sporting goods store. The items she wants to buy are soccer cleats for \$24.99, shin guards for \$12.99, soccer socks for \$4.49, and a soccer ball for \$19.95. How much will the four items cost?
- b. Jada and Tonya ran a 400-meter race. Jada ran the race in 75.2 seconds. Tonya ran the race in 69.07 seconds. How much faster did Tonya run the race?

3. Determine each product.

a. $\frac{3}{8} \times \frac{4}{5}$

b. $2\frac{9}{10} \times \frac{2}{5}$