

Get in Shape

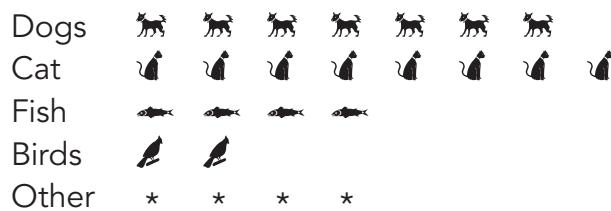
2

Analyzing Numerical Data Displays

WARM UP

Mr. Garcia surveyed his class and asked them what types of pets they owned. Analyze the pictograph that shows the results of his survey:

Pets Owned by Students in Mr. Garcia's Class
(each symbol represents 1 student)



1. How many students in Mr. Garcia's class own dogs?
2. How many students own fish or birds?
3. Can you tell by looking at the pictograph how many students own pets? Why or why not?

LEARNING GOALS

- Create and interpret dot plots.
- Create and interpret stem-and-leaf plots.
- Describe the center, spread, and overall shape of a data distribution.

KEY TERMS

- dot plot
- distribution
- symmetric
- skewed right
- skewed left
- clusters
- gaps
- peaks
- outliers
- stem-and-leaf plot

You know how to use picture graphs, bar graphs, and line plots to display categorical and numerical data. What additional plots can be used to display and analyze numerical data?

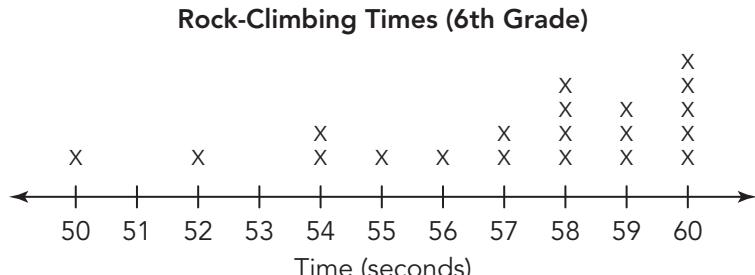
Getting Started

Rock-Climbing Competition

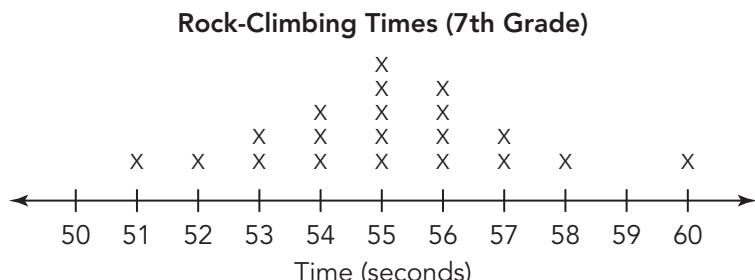
Ms. Nicholson poses the question "Which grade has the fastest average rock-climbing time if each student is given one attempt?"

She selects one class from each grade level, times each student as they climb the rock wall, and records the times. Then she creates data displays for each class.

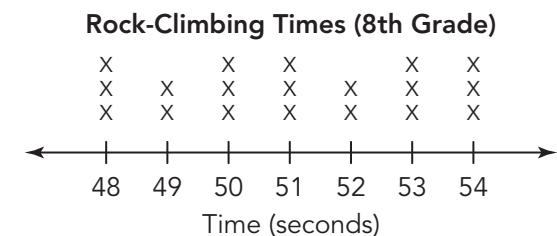
Sixth Grade
Completion Times
(seconds):
60, 50, 58, 59, 60, 54,
55, 58, 59, 60, 52, 54,
56, 57, 57, 58, 60, 60,
59, 58



Seventh Grade
Completion Times
(seconds):
51, 52, 53, 53, 54, 54,
54, 55, 55, 55, 55, 55,
56, 56, 56, 56, 57, 57,
58, 60



Eighth Grade
Completion Times
(seconds):
48, 54, 53, 50, 54,
52, 48, 49, 51, 54, 53,
48, 50, 50, 49, 51, 51,
52, 53



1. How are these data displays similar? How are they different?

2. What can you observe from a data display that you cannot see from looking at the numerical data?



ACTIVITY
2.1

Creating and Analyzing Dot Plots



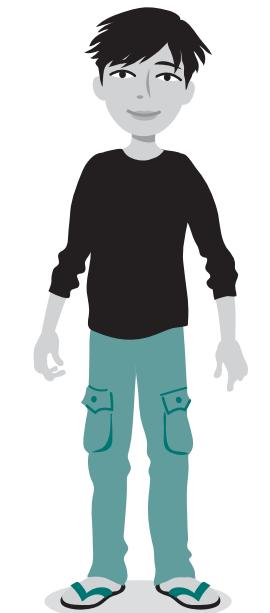
The 2014 Winter Olympics were held in Sochi, Russia. While watching the Olympics, Jessica and Maurice decided to pose statistical questions about the Games.

1. Jessica asked, "How many medals did the United States win? How many of those were gold?" Maurice thought a better set of questions would be, "What is the typical number of medals won? What is the typical number of gold medals won by a country?" Who's correct? Explain your reasoning.



The table at the end of the lesson lists the number of gold medals and the total medals won by all medal-winning countries for the 2014 Winter Olympics.

2. Analyze the data shown in the table.
 - a. What conclusions can you make about the numbers of total medals won at the 2014 Winter Olympics?
 - b. Are the data in the table categorical or quantitative? Explain your reasoning.
 - c. Are the data in the table discrete or continuous? Explain your reasoning.



I wonder if countries who win gold medals tend to win more medals in general.



One way to describe a set of quantitative data is by drawing a graphical display of the data.

A **dot plot** is a data display that shows discrete data on a number line with dots, Xs, or other symbols. Dot plots help organize and display a small number of data points.

WORKED EXAMPLE

This dot plot shows the gold-medal data. The number line represents the number of gold medals. Each X above a number represents the number of countries that won that many gold medals.



3. Use the dot plot to answer each question.

a. What do the two Xs above the number 8 represent?

b. What do the five Xs above the number 0 represent?

c. Why are there no Xs above the number 7?

d. Use the dot plot to determine the number of countries that won medals in the 2014 Winter Olympics. Explain your strategy.

Let's create a dot plot to display the total number of medals won as listed in the 2014 Winter Olympics data table.

4. Make a plan for creating your dot plot.

a. What will you name your dot plot?

b. What numbers will begin and end your number line?

Why did you select these numbers?

c. What interval will you use on your number line? Why did you select this interval?

5. Create your dot plot displaying the data for the total medals won at the 2014 Winter Olympics.

6. Write a brief summary to report the results of your data analysis back to Maurice and Jessica to help answer their questions about gold medals and all medals won at the 2014 Winter Olympics.

These questions are part of analyzing data.



ACTIVITY
2.2

Describing Distributions



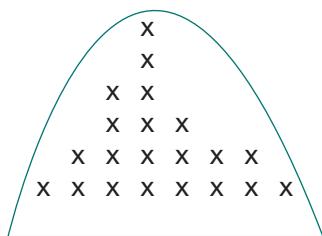
When you analyze a graphical representation of numeric data, you can look at its shape, center, and spread to draw conclusions.

- What is the overall shape of the graph? Does it have any interesting patterns?
- Where is the approximate middle, or center, of the graph?
- What does the graph tell me about how spread out the data values are?

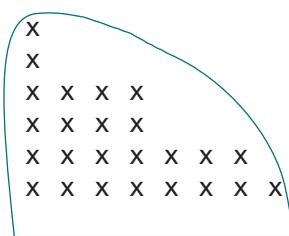
The overall shape of a graph is called the *distribution* of data. A **distribution** is the way in which the data are spread out.

The shape of the distribution can reveal a lot of information about data. There are many different distributions, but the most common are *symmetric*, *skewed right*, and *skewed left*.

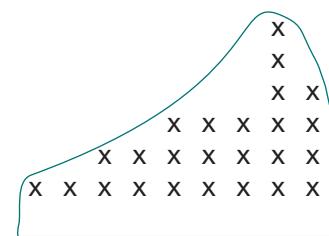
Shapes of Typical Distributions of Graphical Displays of Data



symmetric



skewed right



skewed left

- The left and right halves of the graph are mirror images of each other.
- The peak is in the middle, because there are many data values in the center.

- The peak of the data is to the left side of the graph.
- There are only a few data points to the right side of the graph.

- The peak of the data is to the right side of the graph.
- There are only a few data points to the left side of the graph.

A peak is usually the value with the greatest frequency, or one of the values with the greatest frequency, and is often surrounded by data values with other large numbers of data points.

1. Miko says that the dot plot shown in the previous activity for the number of gold medals won is skewed right. Do you agree with her statement? Explain your reasoning.



Examine the dot plot you created for the total number of medals won by medal-winning countries.

2. What is the distribution of the dot plot? Explain what this means in terms of the total number of medals won.

When analyzing a graphical display of data, you can also look for any interesting patterns. Some of these patterns include:

- **clusters**—areas where data are grouped close together
- **gaps**—areas where there are no data
- **peaks**—values that contain more data points than the values on either side of it
- **outliers**—data values that lie a large distance from the other data. Outliers usually accompany gaps in data.

Gaps usually span multiple possible data values.

Examine the dot plot you analyzed for the number of gold medals won by medal-winning countries.

3. Identify any clusters, gaps, peaks, or outliers. Explain what this means in terms of the number of gold medals won.

Examine the dot plot you created for the total number of medals won by medal-winning countries.

- 4. Identify any clusters, gaps, peaks, or outliers. Explain what this means in terms of the total number of medals won.**

Another common shape for a data distribution is a uniform distribution.

Refer back to the dot plots from the *Rock-Climbing Competition* activity at the beginning of the lesson.

- 5. Describe the shape of each dot plot including its overall shape and any relevant patterns.**



At the 2014 Winter Olympics, 88 countries competed in the events, but only 26 won medals. By contrast, the 2016 Summer Olympics in Rio de Janeiro, Brazil, had 207 countries compete in the events. Athletes from 80 countries won medals, but only 44 won at least 5 medals.

The table at the end of the lesson lists the total number of medals won by the top-performing countries in the 2016 Summer Olympics.

1. What comparisons can you make between the number of medals won at the 2014 Winter Olympics and the number of medals won at the 2016 Summer Olympics?

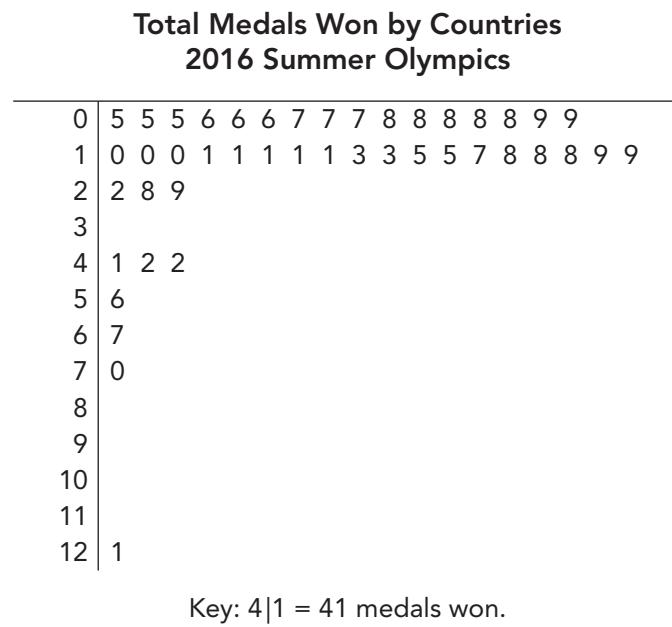
2. Do you think using a dot plot would be a good way to organize and analyze the data in the Summer Olympics table? Explain your reasoning.

A numerical data display that can easily display data sets with a larger range of data values would be helpful to plot the 2016 Summer Olympic data. A **stem-and-leaf plot** is a graphical method used to represent ordered numerical data. Once the data is ordered, the stems and leaves are determined. Typically, the stem is all the digits in a number except the rightmost digit, which is the leaf.

A stem-and-leaf plot displaying the number of medals won in the 2016 Summer Olympics is shown.

3. Use the stem-and-leaf plot to answer each question.

a. Describe what you notice about the stem-and-leaf plot.

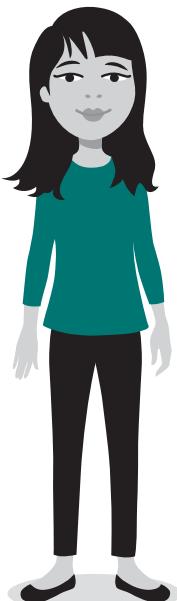


The first column in the plot represents the stems. The numbers after the vertical line are the leaves.

b. What does 7 | 0 mean in the stem-and-leaf plot?

c. What does 0 | 5 mean?

d. How many stems are in the stem-and-leaf plot?



4. Analyze the stems and leaves in the stem-and-leaf plot.

a. How many leaves are in the stem-and-leaf plot?

Why are there that many leaves?

b. Why would a stem have more than one leaf?

c. Why do some stems have no leaves?

d. Why do some stems have the same leaf repeated?



e. Carlos claims that he should write 0s as leaves after the stems 3, 8, 9, 10, and 11 to show that there are no countries that have an amount of medals in the 30s, 80s, 90s, 100s, or 110s. Is Carlos correct? Explain your reasoning.

5. What is the most common number of medals won? How can you determine this from the stem-and-leaf plot?

To see the distribution better, rotate the stem-and-leaf plot so that the stems resemble a horizontal number line.

6. Describe the distribution and any interesting patterns you notice in the stem-and-leaf plot. Interpret your findings in terms of the number of medals won in the 2016 Summer Olympics.

ACTIVITY
2.4

Creating and Analyzing Stem-and-Leaf Plots



During the 2016 presidential election, media reports sometimes called attention to the ages of the candidates. This led to Alicia wondering, "Are these candidates too old to be president?" Because she wanted to collect and analyze data, she revised her question: "At what age do presidents take office?"

1. Explain why Alicia's question is a statistical question.

To answer her question, Alicia collected data on the ages when the 43 former presidents of the United States were first inaugurated. Her data is presented in the table at the end of the lesson.

To analyze the data, let's create a stem-and-leaf plot of the former presidents' ages at inauguration.

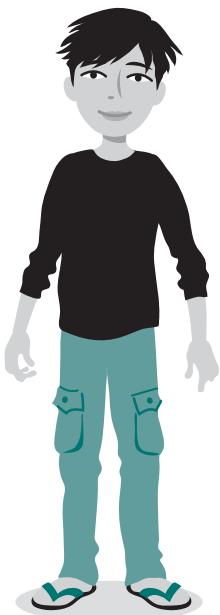
2. Make a plan for creating the stem-and-leaf plot.

a. **What will you choose for your stems? Why did you choose those numbers?**

b. **How many leaves will you have in your stem-and-leaf plot? Explain your reasoning.**

c. **Create a key for your stem-and-leaf plot. Why is this needed?**

Put the data in ascending order, from least to greatest, before filling in the leaves.



3. Create a stem-and-leaf plot to display the age at which each president was inaugurated.
 4. Describe the distribution of the ages of presidents at their inaugurations.
 5. The minimum age to become president of the United States is 35 years old. How is this requirement reflected in your stem-and-leaf plot?
 6. What was the most common age for presidents to be inaugurated? Explain using your stem-and-leaf plot.

The four primary candidates leading into the 2016 presidential election and their projected ages at inauguration are provided.

- 7. How would your stem-and-leaf plot change if each person had been elected?**

Gary Johnson	64
Jill Stein	66
Hillary Clinton	69
Donald Trump	70

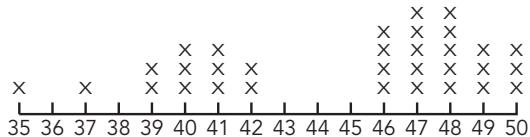
- 8. Write a brief summary to report the results of your data analysis back to Alicia in response to her question about the ages of presidents at their inaugurations.**

TALK the TALK

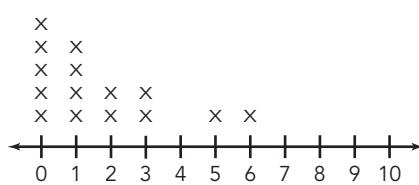
Peaks, Gaps, and Clusters... Oh, My!

Consider the six data displays shown.

Plot A



Plot B

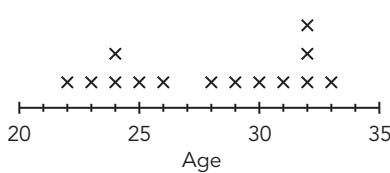


Plot C

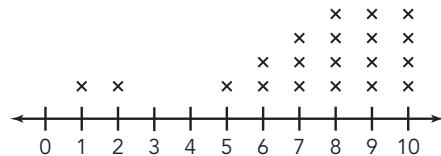
1	9
2	5 5 9
3	0 2 2 2 2 4 4 5 6 6 6 9 9 9
4	0 0 1 1 2 4 5 6 6 7 7 8 9
5	1 1 1 2 3 4 4 6 7
6	9

5|6 means 56

Plot D



Plot E



Plot F

3	2
4	
5	
6	1 4 4
7	2 5 7 9
8	0 2 2 5 6 8
9	0 0 1 2 3 3 5

Key: 3|2 = 3.2

1. Which plot or plots illustrate each graphical feature?
 - a. cluster(s)
 - b. gap(s)
 - c. outlier(s)
 - d. skewness

2. Select a symmetric distribution and explain how you can make it skewed left or skewed right.

Use with Activity 2.1, Creating and Analyzing Dot Plots

2014 Winter Olympics All Countries with Medal Wins					
Country	Gold Medals	Total Medals	Country	Gold Medals	Total Medals
Russian Federation	13	33	Slovenia	2	8
United States	9	28	Japan	1	8
Norway	11	26	Italy	0	8
Canada	10	25	Belarus	5	6
Netherlands	8	24	Poland	4	6
Germany	8	19	Finland	1	5
Austria	4	17	Great Britain	1	4
France	4	15	Latvia	0	4
Sweden	2	15	Australia	0	3
Switzerland	6	11	Ukraine	1	4
China	3	9	Slovakia	1	1
South Korea	3	8	Croatia	0	1
Czech Republic	2	8	Kazakhstan	0	1

Use with Activity 2.3, Stem-and-Leaf Plots

Total Number of Medals Won by Top-Performing Countries, 2016 Summer Olympics			
Country	Medals Won	Country	Medals Won
United States	121	Ukraine	11
China	70	Poland	11
Great Britain	67	Sweden	11
Russian Federation	56	Croatia	10
Germany	42	Czech Republic	10
France	42	South Africa	10
Japan	41	Cuba	9
Australia	29	Belarus	9
Italy	28	Columbia	8
Canada	22	Iran	8
Netherlands	19	Ethiopia	8
Brazil	19	Serbia	8
South Korea	18	Turkey	8
New Zealand	18	Georgia	7
Azerbaijan	18	North Korea	7
Kazakhstan	17	Switzerland	7
Hungary	15	Belgium	6
Denmark	15	Thailand	6
Kenya	13	Greece	6
Uzbekistan	13	Romania	5
Spain	11	Malaysia	5
Jamaica	11	Mexico	5

Use with Activity 2.4, Creating and Analyzing Stem-and-Leaf Plots

President	Age at First Inauguration	President	Age at First Inauguration
Washington	57	Harrison	55
Adams, J.	61	McKinley	54
Jefferson	57	Roosevelt, T.	42
Madison	57	Taft	51
Monroe	58	Wilson	56
Adams, J.Q.	57	Harding	55
Jackson	61	Coolidge	51
Van Buren	54	Hoover	54
Harrison	68	Roosevelt, F.D.	51
Tyler	51	Truman	60
Polk	49	Eisenhower	62
Taylor	64	Kennedy	43
Fillmore	50	Johnson, L.B.	55
Pierce	48	Nixon	56
Buchanan	65	Ford	61
Lincoln	52	Carter	52
Johnson, A.	56	Reagan	69
Grant	46	Bush, G.H.W.	64
Hayes	54	Clinton	46
Garfield	49	Bush, G.W.	54
Arthur	50	Obama	47
Cleveland	47		

Use with the Assignment.

Season	Wins	Losses	Season	Wins	Losses
2015-2016	48	34	1991-1992	38	44
2014-2015	60	22	1990-1991	43	39
2013-2014	38	44	1989-1990	41	41
2012-2013	44	38	1988-1989	52	30
2011-2012	40	26	1987-1988	50	32
2010-2011	44	38	1986-1987	57	25
2009-2010	53	29	1985-1986	50	32
2008-2009	47	35	1984-1985	34	48
2007-2008	37	45	1983-1984	40	42
2006-2007	30	52	1982-1983	43	39
2005-2006	26	56	1981-1982	42	40
2004-2005	13	69	1980-1981	31	51
2003-2004	28	54	1979-1980	50	32
2002-2003	35	47	1978-1979	46	36
2001-2002	33	49	1977-1978	41	41
2000-2001	25	57	1976-1977	31	51
1999-2000	28	54	1975-1976	29	53
1998-1999	31	19	1974-1975	31	51
1997-1998	50	32	1973-1974	35	47
1996-1997	56	26	1972-1973	46	36
1995-1996	46	36	1971-1972	36	46
1994-1995	42	40	1970-1971	36	46
1993-1994	57	25	1969-1970	48	34
1992-1993	43	39	1968-1969	48	34

Assignment

Write

Write a definition for each of term in your own words.

- | | |
|-----------------|------------------------|
| 1. dot plot | 6. clusters |
| 2. distribution | 7. gaps |
| 3. symmetric | 8. peaks |
| 4. skewed right | 9. outliers |
| 5. skewed left | 10. stem-and-leaf plot |

Remember

Data sets have distributions that can be described according to their shape.

Dot plots are ideal for small data sets.

Stem-and-leaf plots are ideal for moderately sized data sets, especially if you need to see the actual data values.

Practice

The data table at the end of the lesson shows the number of wins and losses the Atlanta Hawks have had in 48 seasons in Atlanta.

1. Create a dot plot or a stem-and-leaf plot for the number of wins by the Atlanta Hawks. Be sure to name your plot and provide a key if necessary.
2. Describe the distribution of the data. Include any specific graphical features or patterns. Explain what your answer means in terms of the number of wins by the Hawks.
3. Create a dot plot or a stem-and-leaf plot for the number of losses by the Atlanta Hawks. Be sure to name your plot and provide a key if necessary.
4. Describe the distribution of the data. Include any specific graphical features or patterns. Explain what your answer means in terms of the number of losses by the Hawks.
5. Propose a win-loss record for an upcoming season that would result in a change in the overall distribution of both plots.

Stretch

Another type of display used to compare two data sets is a side-by-side or back-to-back stem-and-leaf plot.

1. Describe the distribution of each data set.
2. Then, use the key and the plot to list the numerical data values in each data set.

Data Set One	Data Set Two
8 6	4
9 3 1	5 9
9 8 6 5 1 1 0	6 2 7
7 3 2	7 0 0 3 6 6 8 9
5 3	8 0 1 1 2
2 1	9

Key: 1|5|9 = 5.1 and 5.9

Review

1. Write a statistical question about each situation.
 - a. vacation destinations
 - b. books
2. Plot and identify 4 points on a coordinate plane that are vertices of a parallelogram. Include points in more than one quadrant. Draw the parallelogram. Write absolute value statements for the length of the base and height of your parallelogram. Then, determine the area of the parallelogram.
3. Use long division to determine each quotient.
 - a. $247 \div 8$
 - b. $894 \div 12$