# 3

# Oh, Yes, I Am the Muffin Man

## Determining Equivalent Ratios

## WARM UP

Choose the correct statement to complete each sentence and explain your reasoning.

1. When the manager at Sweets-a-Plenty Bakery decides how many bakers are needed to bake muffins for a given day, she needs to consider the total number of muffins needed for the day.

   a. Making fewer muffins with more bakers will take:
      - less time.
      - an equal amount of time.
      - more time.

   b. Making more muffins in a shorter amount of time requires:
      - fewer workers.
      - an equal amount of workers.
      - more workers.

## LEARNING GOALS

- Use drawings to model and determine equivalent ratios.
- Reason about tape diagrams to model and determine equivalent ratios.
- Define and use rates and rate reasoning to solve ratio problems.
- Use scaling up and scaling down to determine equivalent ratios.
- Use double number lines to solve real-world problems involving ratios.

## KEY TERMS

- equivalent ratios
- tape diagram
- rate
- proportion
- scaling up
- scaling down
- double number line

Informally comparing ratios, or qualitatively comparing ratios, is important. However, there are many instances when you need to make more specific comparisons. How can you use equivalent ratios in order to compare ratios more precisely?

# Which Has More?

Consider the given representations to answer each question. Explain your reasoning.

**1. Which dinner order has more pizza?**

Order 1

Order 2

**2. Which pattern has more stars?**

Pattern 1

Pattern 2

**3. Which pile of laundry has more shirts?**

Pile 1

Pile 2

**4. Which type of reasoning did you use for each question—additive or multiplicative? Explain why.**

ACTIVITY
3.1

# Using Drawings to Model Equivalent Ratios

Kerri and her friends are going hiking. Kerri invites her friends to meet at her house for a quick breakfast before heading out on their hike. Kerri wants to offer muffins to her friends.

**1. She knows that one muffin combo has four muffins that can feed four people.**

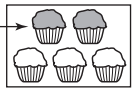
**a. Draw a model showing the relationship between the muffin combo and the number of people it will feed.**

**b. If Kerri invites 6 friends, how many muffin combos will she need? Draw a model to show how many muffin combo(s) she will need, and explain your answer.**

**c. If Kerri has $2\frac{3}{4}$ muffin combos, how many friends can she feed? Draw a model to show how many friends she can feed, and explain your answer.**

Let's consider a different variety pack.

In one muffin variety pack, two out of every five muffins are blueberry.

blueberry muffin 

**2. Draw a model to answer each question. Explain your reasoning.**

**a. How many muffins are blueberry muffins if there are a total of 25 muffins?**

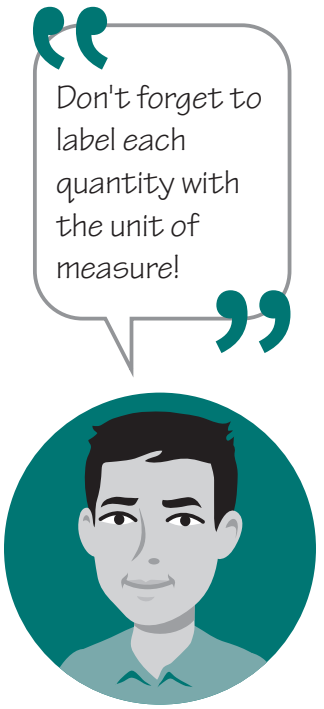

**b. How many muffins are blueberry muffins if there are a total of 35 muffins?**

**c. How many total muffins are there if 8 muffins are blueberry?**

As you solved these problems, you determined *equivalent ratios*. **Equivalent ratios** are ratios that represent the same part-to-part or part-to-whole relationship.

ACTIVITY
# 3.2 Tape Diagrams

The local bakery sells muffins in variety packs of blueberry, pumpkin, and bran muffins. They always sell the muffins in the ratio of 3 blueberry muffins : 2 pumpkin muffins : 1 bran muffin.




**1. Write the ratio that expresses each relationship. Identify each as a part-to-part or a part-to-whole ratio.**

**a. blueberry muffins to total muffins**

**b. pumpkin muffins to total muffins**

**c. bran muffins to total muffins**

**d. blueberry muffins to pumpkin muffins**

**e. bran muffins to pumpkin muffins**

**f. blueberry muffins to bran muffins**

A ratio can be represented by drawing the objects themselves, but they also can be represented using a *tape diagram*. A **tape diagram** illustrates number relationships by using rectangles to represent ratio parts. A tape diagram representing the ratio of each type of muffin is shown.


**2. What does each small rectangle represent in the given tape diagram?**

Remember, in this scenario the ratio of muffins in each variety pack is always 3 blueberry muffins : 2 pumpkin muffins : 1 bran muffin.

Tape diagrams provide a visual representation of ratios, but they also can be used to solve problems.

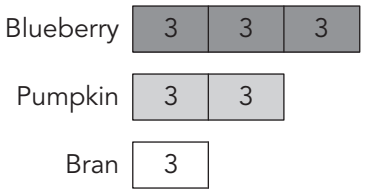
**WORKED EXAMPLE**

Suppose you purchase an 18-pack of muffins. How many blueberry, pumpkin, and bran muffins will you purchase?

There are 6 muffins represented in the tape diagram, and you want 18 total muffins that are in the same ratio.

Therefore, to determine how many muffins you need to maintain the same ratio, you can divide 18 by 6.
$18 \div 6 = 3$

Therefore, each rectangle will represent 3 muffins.




From the tape diagram, you can see that there are 9 blueberry muffins, 6 pumpkin muffins, and 3 bran muffins.

**3. Is the ratio 9 : 6 : 3 equivalent to 3 : 2 : 1? Explain how you know.**

NOTES

4. **Suppose you purchase a 36-pack of muffins. Use the tape diagram to illustrate how many blueberry, pumpkin, and bran muffins you will receive.**


5. **Suppose you wanted 20 pumpkin muffins in your variety pack. How many total muffins will be in your variety pack? Complete the tape diagram to determine the answer.**

6. The table shows the number of muffins in specific sized variety packs. Complete just the missing cells in the columns for the 6-pack and 36-pack of muffins.

| Total Number of Muffins | 6 | 12 | 18 | 24 | 36 |
|---|---|---|---|---|---|
| Number of Blueberry Muffins | | | 9 | | |
| Number of Pumpkin Muffins | | | 6 | | |
| Number of Bran Muffins | | | 3 | | |

7. Analyze the completed columns in the table.

   a. What do you notice about the numbers?

   b. How could you have determined the number of each type of muffin in the 18-pack without using the tape diagram?

   c. How could you have determined the number of each type of muffin in the 36-pack without using the tape diagram?

   d. Use what you noticed about the numbers in the table to complete the remaining columns for the number of each type of muffin in a 12-pack and in a 24-pack of muffins. Explain your strategy.

ACTIVITY
# 3.3 Rates and Proportions

One of the rounds at the Math Quiz Bowl tournament is a speed round. A team of four students will represent Stewart Middle School in the speed round of the Math Quiz Bowl. One student of the team will be chosen to solve as many problems as possible in 20 minutes. The results from this week's practice are recorded in the table.

| Student | Number of Correctly Solved Problems in a Specified Time |
|---|---|
| Kaye | 4 problems correct in 5 minutes |
| Susan | 7 problems correct in 10 minutes |
| Doug | 1 problem correct in 2 minutes |
| Mako | 3 problems correct in 4 minutes |

**1. Explain how Tia's reasoning and Lisa's reasoning about who should compete in the speed round are incorrect.**

### Tia

Susan should definitely compete in the speed round because she correctly solved the most problems.

### Lisa

It took Susan the longest time to complete her problems. She should not compete in the speed round.

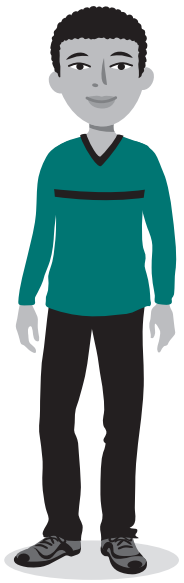
Each quantity in the table is a *rate*. A **rate** is a ratio that compares two quantities that are measured in different units. The rate for each student in this situation is the number of problems solved per amount of time.

**WORKED EXAMPLE**

Kaye's rate is 4 problems correct per 5 minutes. This rate can be written as:

$$\frac{4 \text{ problems correct}}{5 \text{ minutes}}.$$

"It's important to line up the units when writing equal ratios."



**2. Write the rates for the other three team members.**

**a. Susan** **b. Doug** **c. Mako**

When two ratios or rates are equivalent to each other, you can write them as a *proportion*. A **proportion** is an equation that states that two ratios are equal. In a proportion, the quantities composing each part of the ratio have the same multiplicative relationship between them.

**WORKED EXAMPLE**

For example, you know that Kaye got four problems correct per 5 minutes. So, you can predict how many problems she could answer correctly in 20 minutes.

$$\frac{\text{problems correct}}{\text{minutes}} \rightarrow \frac{4}{5} = \frac{\boxed{16}}{20}$$

($\times 4$ above and below)

Kaye can probably answer 16 problems correctly in 20 minutes.

When you change one ratio to an equivalent ratio with larger numbers, you are *scaling up* the ratio. **Scaling up** means you multiply both parts of the ratio by the same factor greater than 1.

**3. Use the definition of a ratio to verify that $\frac{4}{5}$ is equivalent to $\frac{16}{20}$.**

Remember, one way to represent a ratio is in fractional form. It doesn't matter which quantity is in the numerator or denominator; it matters that the unit of measure is consistent among the ratios.

**WORKED EXAMPLE**

You can write the proportion in a different way.

$$\frac{\text{minutes}}{\text{problems correct}} \quad \frac{5}{4} = \frac{20}{16} \quad (\times 4,\ \times 4)$$

**4. Determine the number of problems each student can probably solve in 20 minutes. Explain the scaling up you used to determine the equivalent ratio.**

**Susan** **Doug** **Mako**


**5. Which team member is the fastest? Who would you pick to compete? Explain your reasoning.**

ACTIVITY
# 3.4
# Scaling Up and Scaling Down

The muffin variety packs baked by the Healthy for U Bakery come in a ratio of 2 blueberry muffins to 5 total muffins.

**1. Scale up each muffin ratio to determine the unknown quantity.**

a. $\frac{\text{2 blueberry muffins}}{\text{5 total muffins}} = \frac{\text{20 blueberry muffins}}{\text{? total muffins}}$

b. $\frac{\text{2 blueberry muffins}}{\text{5 total muffins}} = \frac{\text{30 blueberry muffins}}{\text{? total muffins}}$

c. $\frac{\text{2 blueberry muffins}}{\text{5 total muffins}} = \frac{\text{? blueberry muffins}}{\text{100 total muffins}}$

d. $\frac{\text{2 blueberry muffins}}{\text{5 total muffins}} = \frac{\text{50 blueberry muffins}}{\text{? total muffins}}$

e. $\frac{\text{2 blueberry muffins}}{\text{5 total muffins}} = \frac{\text{? blueberry muffins}}{\text{15 total muffins}}$

f. $\frac{\text{2 blueberry muffins}}{\text{5 total muffins}} = \frac{\text{28 blueberry muffins}}{\text{? total muffins}}$

When you change a ratio to an equivalent ratio with smaller numbers, you are *scaling down* the ratio. **Scaling down** means you divide both parts of the ratio by the same factor greater than 1, or multiply both parts of the ratio by same factor less than 1. Scaling down a ratio often makes it easier to understand.

Remember the definition of division, $a \div b = a \cdot \frac{1}{b}$.

**2. Scale down each ratio to determine the unknown quantity.**

a. $\frac{\text{3 people}}{\text{9 pizzas}} = \frac{?}{\text{3 pizzas}}$

b. $\frac{\text{2 hoagies}}{\text{6 people}} = \frac{\text{1 hoagie}}{?}$

c. $\frac{\text{100 track shirts}}{\text{25 people}} = \frac{?}{\text{1 person}}$

d. $\frac{\text{60 tracks}}{\text{5 CDs}} = \frac{?}{\text{1 CD}}$

e. $\frac{\text{3 tickets}}{\$26.25} = \frac{\text{1 ticket}}{?}$

f. $\frac{\text{12 hours}}{\text{720 miles}} = \frac{\text{4 hours}}{?}$

g. $\frac{\text{20 hours of work}}{\$240} = \frac{\text{1 hour of work}}{?}$

h. $\frac{\text{3 gallons of red paint}}{\text{2 gallons of yellow paint}} = \frac{?}{\text{1 gallon of yellow paint}}$

ACTIVITY
# 3.5 Double Number Lines

An interval is the amount of space between two tick marks on a number line.

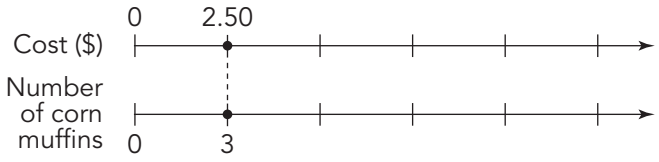
You know several strategies to determine the relationship between two quantities: drawing models, building tape diagrams, and scaling up or down. You can also use a *double number line* to visualize these relationships. A **double number line** is a model that is made up of two number lines used together to represent the ratio between two quantities. The intervals on each number line maintain the same ratio.

The Muffin Man Bakery offers two types of muffins—corn or cinnamon raisin. It costs the bakery $2.50 to make 3 corn muffins.

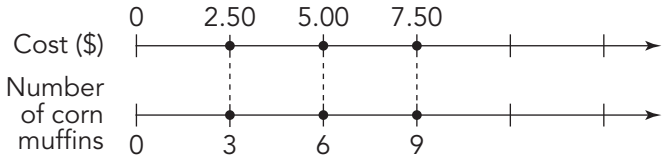
The scale for each number line is different, but the interval is the same for both lines.

## WORKED EXAMPLE

The ratio $2.50 : 3 corn muffins is shown on the double number line.




You can see other equivalent ratios of *cost : number of corn muffins* by continuing to label each interval.

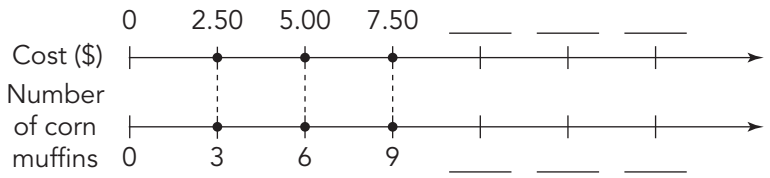



**1. State the two new ratios of cost : number of corn muffins shown on the second double number line.**

2. Describe the interval represented on each number line.

3. Use the double number line to determine equivalent ratios.

   a. Plot the new ratios. Explain your calculations.



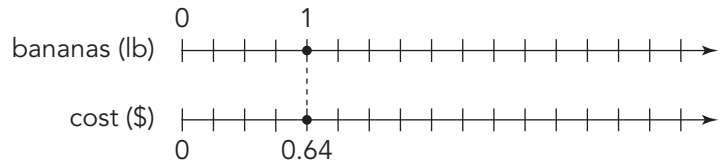

   b. What is the cost of making 12 corn muffins?

   c. What is the cost of making 15 corn muffins?

   d. What is the cost of making 18 corn muffins?

   e. Describe any patterns you notice between the cost and the number of corn muffins made.

4. One pound of bananas costs $0.64. Use the double number lines to determine the cost for each quantity of bananas.

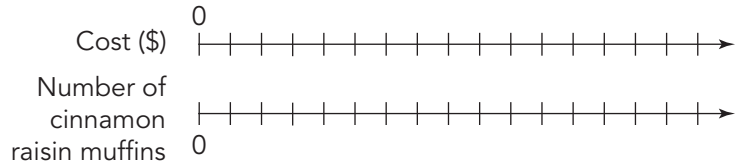



a. $2\frac{1}{2}$ pounds

b. $\frac{1}{2}$ pound

c. 2 pounds

5. The cost for The Muffin Man Bakery to make 4 cinnamon raisin muffins is $3.20. Use the double number line to determine equivalent ratios and answer each question. Explain your calculations.

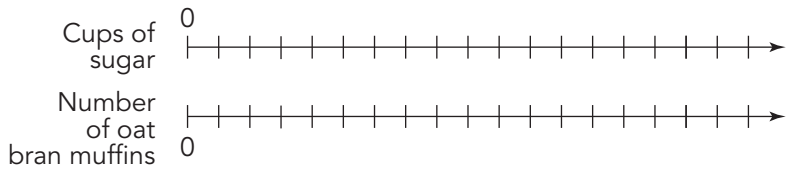



a. What is the cost to make 8 cinnamon raisin muffins?

b. How many cinnamon raisin muffins are made for $12.80?

c. What is the cost of making 12 cinnamon raisin muffins?

**6. It takes 1 cup of sugar to make 12 oat bran muffins. Use the double number line to determine equivalent ratios and answer each question. Explain your calculations.**




**a. Plot the given ratio on the double number line.**

**b. How many oat bran muffins can be made using $\frac{1}{2}$ cup of sugar? $\frac{2}{3}$ cup of sugar? $1\frac{1}{2}$ cups of sugar?**

**c. How many cups of sugar are needed to make 3 muffins? 15 muffins? 9 muffins?**

NOTES

## TALK the TALK

### Make a Choice

Answer each question by using pictures, a tape diagram, or a double number line. Show all of your work and explain why you chose your strategy.

1. **A T-shirt store keeps 7 white T-shirts on the shelves for every 3 purple T-shirts on the shelves.**

   a. **How many white T-shirts are on the shelves if there are 15 purple T-shirts on the shelves?**

   b. **How many purple T-shirts are on the shelves if there are 49 white T-shirts on the shelves?**

   c. **How many white shirts are on the shelves if there are 40 total shirts (purple and white) on the shelves?**

2. **A grocery store advertises 4 pounds of apples for $6.00.**

   a. **What is the cost for 3 pounds of apples?**

   b. **What is the cost for 1 pound of apples?**

   c. **How many pounds of apples can you purchase with $40.00?**

Circle the question that your teacher has asked you to present to the class. Write at least 3 sentences to tell your classmates how you completed the work.

# Assignment

## Write

Compare and contrast tape diagrams and double number line models for representing ratio relationships. Use an example in your description.

## Remember

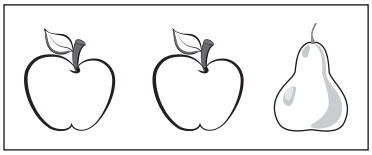
Equivalent ratios are ratios that represent the same part-to-part or part-to-whole relationship.

A proportion is an equation that states that two ratios are equal. In a proportion, the quantities composing each part of the ratio have the same multiplicative relationship between them.

Scaling up means you multiply both parts of the ratio by the same factor greater than 1.

Scaling down means you divide both parts of the ratio by the same factor greater than one, or multiply both parts of the ratio by the same factor less than 1.

## Practice



1. Ms. Yoto is putting together bags of fruit that contain 1 pear for every 2 apples. For each ratio given, create a picture module. Then, calculate the answer from your model, and explain your reasoning.
   a. How many apples are in the bag if there are a total of 9 pieces of fruit?
   b. How many apples are in the bag if there are a total of 15 pieces of fruit?
   c. How many pieces of fruit are there if there are 8 apples in the bag?
2. When creating playlists for dances, DJ Lew likes to maintain a ratio of 4 hip hop songs : 3 country songs : 1 slow song.
   a. Create a tape diagram to represent this ratio.
   b. Suppose DJ Lew has 40 songs on his playlist. Use the tape diagram to illustrate how many hip hop, country, and slow songs are on the playlist.
   c. Suppose DJ Lew wants to put 36 hip hop songs on the playlist. How many total songs will be on the playlist? Use a tape diagram to determine the answer.
3. Scale up or scale down each ratio to complete the proportion.
   a. $\frac{2 \text{ teachers}}{26 \text{ students}} = \frac{8 \text{ teachers}}{?}$
   b. $\frac{12 \text{ inches}}{1 \text{ foot}} = \frac{?}{18 \text{ feet}}$
   c. $\frac{\$39{,}000}{1 \text{ year}} = \frac{?}{3 \text{ years}}$
   d. $\frac{18 \text{ pencils}}{1 \text{ box}} = \frac{108 \text{ pencils}}{?}$
   e. $\frac{\$40}{15 \text{ gallons}} = \frac{?}{3 \text{ gallons}}$
   f. $\frac{1200 \text{ boxes}}{9 \text{ truckloads}} = \frac{?}{3 \text{ truckloads}}$
   g. $\frac{280 \text{ beats}}{4 \text{ seconds}} = \frac{70 \text{ beats}}{?}$
   h. $\frac{520 \text{ cm}}{5.2 \text{ m}} = \frac{260 \text{ cm}}{?}$

4. A mason is a person who builds structures with bricks, stone, cement block, or tile. A mason usually uses mortar to hold the bricks together. A general rule of thumb in masonry is that $2\frac{1}{2}$ bags of mortar are needed for every 100 cement blocks.
   a. Complete a double number line to determine the amount of mortar needed for each quantity of blocks.
   b. How many bags of mortar will a mason need for 350 blocks?
   c. How many bags of mortar will a mason need for 50 blocks?
   d. With $12\frac{1}{2}$ bags of mortar, how many blocks can the mason lay?

## Stretch

Scale up or scale down each ratio to complete the proportion.

1. $\frac{\text{7 cups of red dye}}{\text{10 cups of yellow dye}} = \frac{\text{?}}{\text{25 cups of yellow dye}}$

2. $\frac{\text{?}}{\text{175 in.}} = \frac{\$42}{\text{50 in.}}$

3. $\frac{\text{47 feet}}{\text{60 seconds}} = \frac{\text{?}}{\text{45 seconds}}$

## Review

1. In planning for the upcoming regional girls' tennis tournament, Coach McCarter looked at her players' statistics from the previous 2 months.

   Sarah: 7 matches won, 3 matches lost

   Sophie: 6 matches won, 4 matches lost

   Grace: 7 matches won, 4 matches lost

   Based on their records, which player should Coach McCarter choose to attend the regional tournament? Explain your reasoning.

2. Hydrate sports drink calls for 7 scoops for every gallon of water. Sarah thinks the drink is too weak, and she wants to change it. Describe how she can change either the number of scoops or the amount of water to make the drink stronger.

3. Decide whether each amount is more closely related to volume or surface area.
   a. the amount of air in a room
   b. I recommend changing the example since a hamster cage is made of wire, not sheet metal.

4. Determine each product.
   a. $\frac{2}{5} \times \frac{7}{3}$
   b. $4\frac{1}{6} \times 3\frac{4}{5}$