5

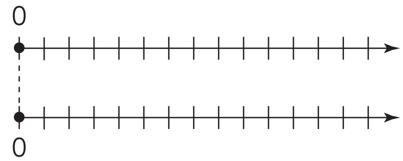
They're Growing!

Graphs of Ratios

WARM UP

A tree grows at a constant rate of 3 feet per year.

1. Write a ratio to represent the amount of growth in feet : the number of months.
2. Create a double number line that describes the growth of the tree every 12 months over a 48-month period.



LEARNING GOALS

- Plot ratios and equivalent ratios on a coordinate plane.
- Read equivalent ratios from graphs.
- Use ratio reasoning to determine equivalent ratios from graphs.
- Recognize the graphical representation of equivalent ratios.

Key Term

- linear relationship

So far, you have used scaling up or scaling down, tables, tape diagrams, pictures, and double number lines to determine equivalent ratios. How can you plot pairs of values on a coordinate plane and determine equivalent ratios?

Getting Started

Growing Rectangles

Consider a rectangle with a short side of length 2 units and a long side of length 3 units.

- In the first table, add the indicated number of units to both the long and short sides of the original rectangle.
- In the second table, multiply each original side length by the given value.
- For each rectangle, determine the ratio of the long side length : short side length.

	Original	+2 units	+3 units	+4 units
Long side	3			
Short side	2			
Ratio	3 : 2			

	Original	×2 units	×3 units	×4 units
Long side	3			
Short side	2			
Ratio	3 : 2			

1. What do you notice about the ratios for rectangles formed by adding to the sides of the rectangle?

2. What do you notice about the ratios for rectangles formed by multiplying the sides of the rectangle by a given value?

ACTIVITY 5.1

Analyzing Rectangle Ratios

You have 2 copies of Rectangle A. You need both for Question 6.

Analyze the rectangles at the end of the lesson.

1. Cut out each rectangle and sort into at least two piles. Share your sorts and your criteria.

2. Determine the side lengths of each rectangle. Label each rectangle with the length of its short side and the length of its long side.

3. Ava grouped together Rectangles A, C, E, F, G, and J. What do you think was her reasoning?

4. Gabriel's sort was similar to Ava's but he included Rectangle A with Rectangles B, D, H, I, and K. What do you think was his reasoning?

5. Complete the table for Ava's Group and Gabriel's Group. Write the ratios in fractional form, comparing the length of the short side to the length of the long side. Compare the ratios in each table. What do you notice?

Ava's Group

	Short	Long	Ratio
A			
C			
E			
F			
G			
J			

Gabriel's Group

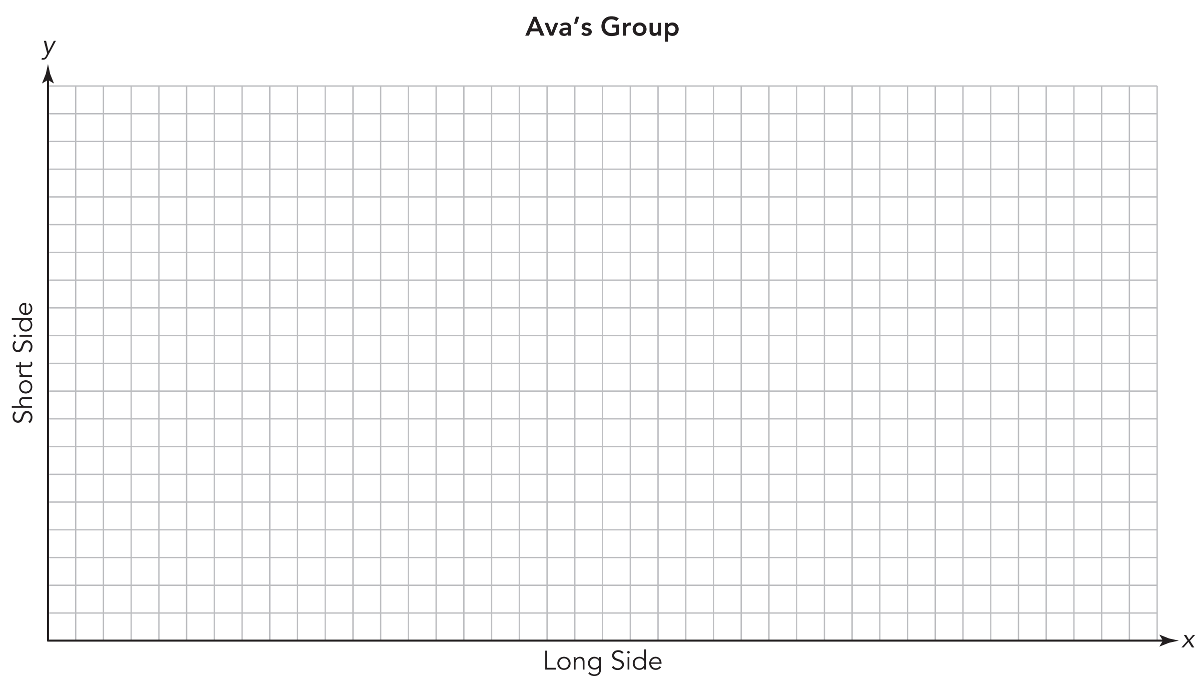
	Short	Long	Ratio
A			
B			
D			
H			
I			
K			

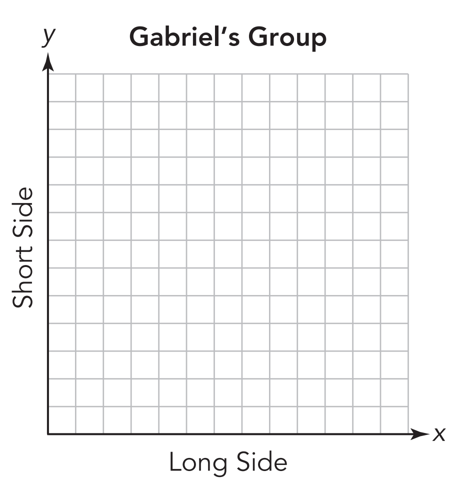
6. Stack each group of rectangles with the smallest rectangle on top so that their longer sides are horizontal and their lower left corners align. What do you notice?

   a. Ava's Group

   b. Gabriel's Group

7. Attach each set of stacked rectangles to the appropriate coordinate grid, with the lower left corner of the rectangles at the origin of the grid.





8. **Label the coordinates of the upper right corner of each rectangle. What do you notice about the coordinates in relation to your ratio?**

9. **Draw a line through the labeled points on each graph. What do you notice about which ordered pairs each line passes through?**

When a set of points graphed on a coordinate plane forms a straight line, a **linear relationship** exists.

Just as equivalent ratios can be represented using tables and double number lines, they can also be represented on the coordinate plane. The ratio $\frac{y}{x}$ is plotted as the ordered pair (x, y). When you connect the points that represent the equivalent ratios, you form a straight line that passes through the origin, such as with Ava's Group. In contrast, non-equivalent ratios are those represented by points that do not create a straight line through the origin, like Gabriel's Group.

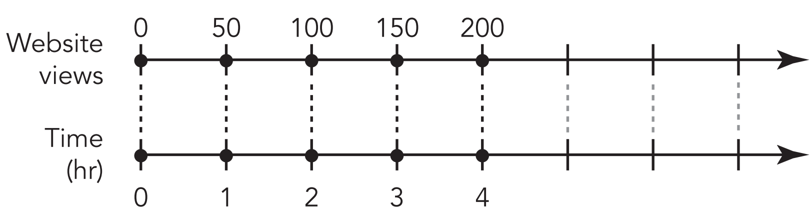
ACTIVITY 5.2 Graphing Equivalent Ratios

Let's investigate how you can use a graph to determine other equivalent ratios, and see how all the representations are connected.

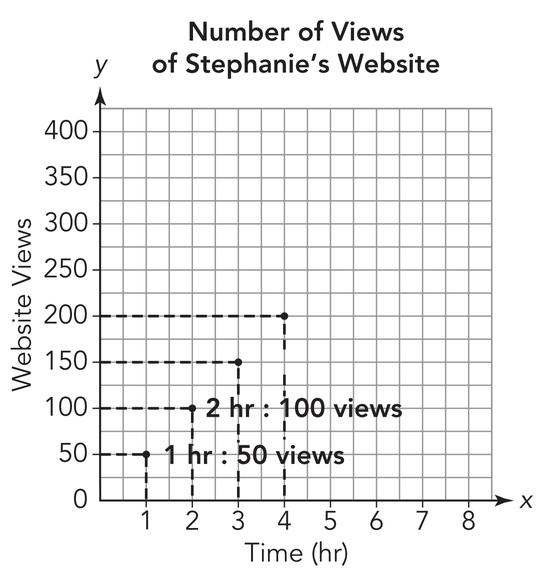
Stephanie runs a website for a local sports team that gets 50 views every hour. The table shows the ratio *time* : *website views.*

Website Views	50	100	150	200
Time (hr)	1	2	3	4

The double number line shown represents the same data.



You can also represent equivalent ratios on a coordinate plane.



1. Label the remaining ratios on the graph.

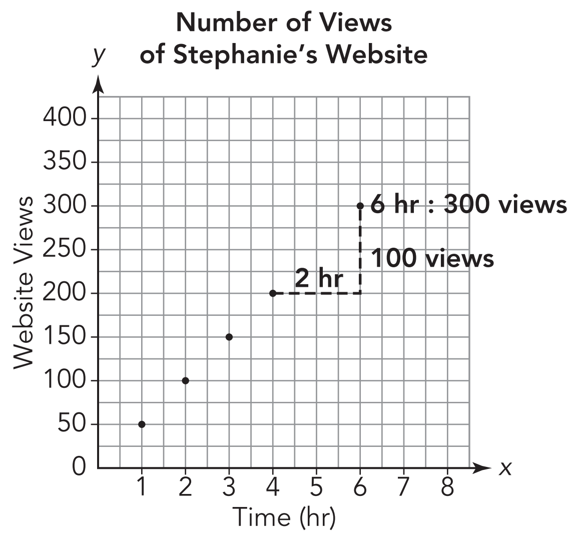
You have used various strategies to determine equivalent ratios:

- drawing pictures
- tape diagrams
- scaling up or down
- ratio tables, and
- double number lines.

WORKED EXAMPLE

Consider the question: How many views will Stephanie's website have in 6 hours?

You know 4 different equivalent ratios from the original graph. The graph shows how to use the two ratios 2 hr : 100 views and 4 hr : 200 views to determine the equivalent ratio 6 hr : 300 views.



Stephanie's website will have 300 views in 6 hours.

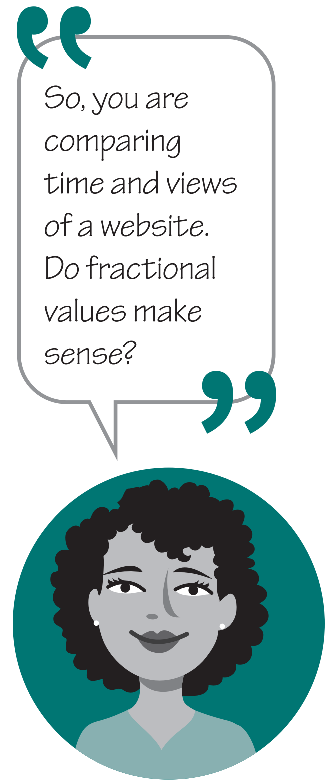
2. Describe how to determine how many views Stephanie's website will have in 7 hours given each representation.

a. using the graph

b. using the table

c. using the double number lines

One way to analyze the relationship between equivalent ratios displayed on a graph is to draw a line to connect the points. You can also extend the line to make predictions of other equivalent ratios. Sometimes, all of the points on the line make sense. Other times when you draw a line, not all the points on the line make sense.

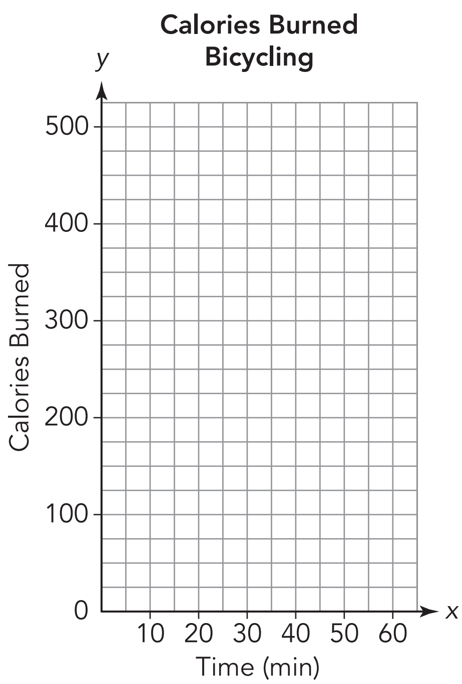


3. Draw a line through all the points you plotted on your graph. Do all the points on the line you drew make sense in this problem situation? Why or why not?

4. How do all the representations—tables, double number lines and graphs—show equivalent ratios? How are they similar? Describe some of the advantages of each representation.

ACTIVITY 5.3 Using Ratio Graphs to Solve Problems

Augie burns 225 calories for every 30 minutes he rides his bike.

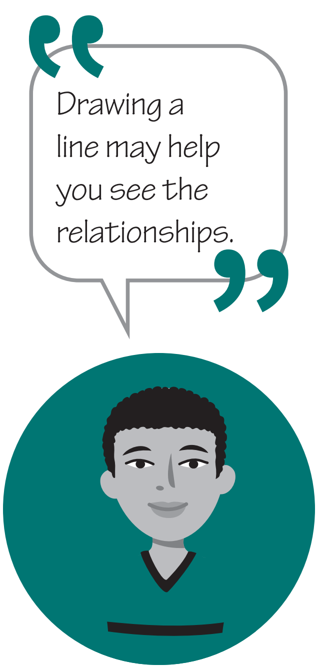


1. Complete the table to chart the number of calories burned for different amounts of time. Then plot the table of values on the graph.

Calories Burned				
Time (min)	30	10	60	50

2. Use your graph to answer each question.

a. How many minutes would Augie have to bike to burn 150 calories?



b. How many calories can he burn if he bikes for 25 minutes?

3. How was the graph helpful? Were there any limitations when using the graph to determine values?

NOTES

TALK the TALK

To Graph or Not to Graph

Go back and examine all the graphs in this lesson.

1. **What is similar about all of the graphs?**

2. **What is different about all the graphs?**

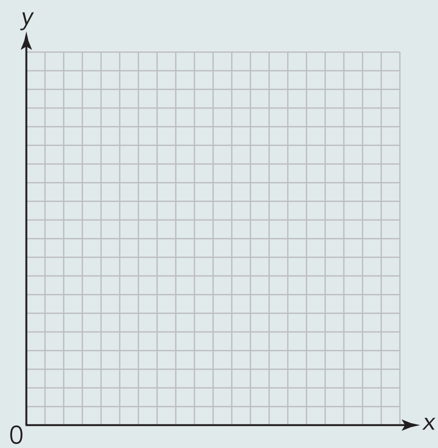
3. **Describe how you can use a line to analyze equivalent ratios. What are the benefits and limitations of using a graph to display and interpret ratios?**

4. Complete the graphic organizer to demonstrate your understanding of ratios.

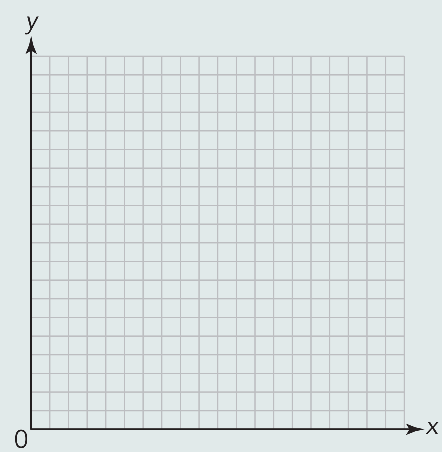
DEFINITION

CHARACTERISTICS

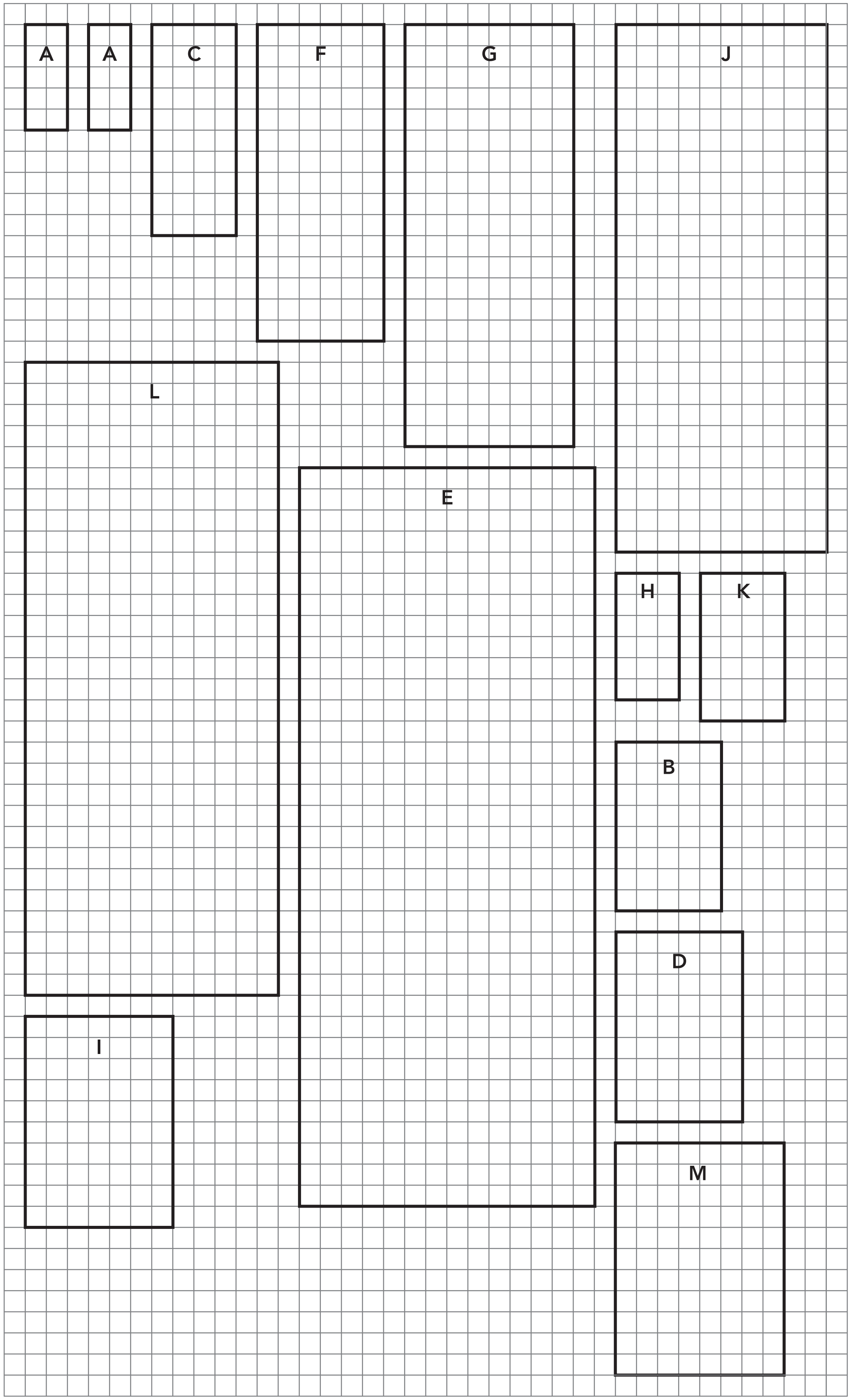
RATIO



EXAMPLE



NON-EXAMPLE


A
A
C
F
G
J
L
E
H
K
B
D
I
M

Assignment

Write

Compare the graph of a ratio relationship with the graph of a relationship that is not represented by a ratio. How are they similar and different? Use an example to explain.

Remember

Just as equivalent ratios can be represented using tables and double number lines, they can also be represented in the coordinate plane. The ratio $\frac{y}{x}$ is plotted as the ordered pair (x, y).

When you connect the points that represent the equivalent ratios, you form a straight line that passes through the origin. In contrast, non-equivalent ratios are those represented by points that cannot be connected by a straight line through the origin.

Practice

Create a graph to represent the values shown in each ratio table.

1.

Weight (pounds)	1	2	4	5
Cost (dollars)	3	6	12	15

2.

Time (hours)	1	3	5	7
Distance (miles)	25	75	125	175

3.

Time (minutes)	15	30	45	60
Calories	80	160	240	320

4.

Time (seconds)	1	10	15	20
Data (Mb)	10	100	150	200

5.

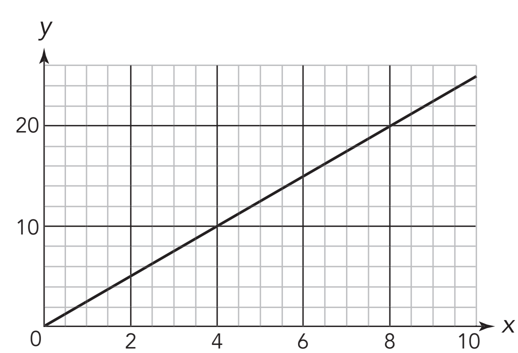
Time (minutes)	15	30	45	60
Distance (miles)	1.5	3	4.5	6

6.

Time (minutes)	1	5	6	10
Height (feet)	6	30	36	60

Stretch

Create a scenario that could be represented by the relationship on the given graph. Describe the quantities, label the axes, and identify at least 4 equivalent ratios.



Review

1. Ellen loves to make her own clothes. With 45 yards of cloth, she can make 5 dresses. Create a double number line to explain your reasoning for each question.
   a. If Ellen has 72 yards of cloth, how many dresses can she make?
   b. If Ellen is going to make a dress for herself, how many yards of cloth does she need?

2. A customer used a $10 bill to pay for a 39-cent candy bar. Simone returned 61 cents. What mistake did Simone make? Explain how she should correct her mistake.

3. A grocery store is selling ground beef for $1.89 per pound. How much does it cost to buy 2.5 pounds?

4. Use estimation to place the decimal point in the correct position in each quotient.

   a. $2.1\overline{)48.72} = 232$

   b. $8\overline{)204.8} = 256$